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Problem 1 (Durable Consumption)

Consider a 2-period model with durable consumption goods. In this model, the durable consumption goods in period t continue to provide utility in period $t+1$. Therefore, the service flow households received from the consumption in period $t+1$ depends on the durable consumption in period t. In addition, durable consumption goods are assumed to depreciate at the rate $\delta \in (0,1)$. The following utility function describes household preferences:

$$U(C_t, C_{t+1}) = \ln(C_t) + \beta \ln(C_{t+1} + (1-\delta)C_t),$$

where C_t and C_{t+1} are the consumption in periods t and $t+1$, respectively. β is the discount factor. Households receive endowments Y_t in period t and Y_{t+1} in period $t+1$. Denote B_{t+1} as the saving (or borrowing) from period t with a fixed interest rate given by r. Assume $B_t = B_{t+2} = 0$.

1. (5 points) Write down the household's budget constraints in period t and $t+1$, respectively.
2. (5 points) Combine the two constraints into an intertemporal budget constraint.
3. (5 points) Solve for the equilibrium C_t and C_{t+1}.
4. (5 points) Solve for B_{t+1}.
5. (5 points) Find the condition for δ such that B_{t+1} moves in the opposite direction of Y_t.
6. (5 points) Now suppose that $\delta = 1$, i.e., consumption is not durable. What is the sign of $\frac{\partial B_{t+1}}{\partial Y_t}$?

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Problem 2 (Solow Model)

Consider a Solow model where households require a subsistence level of consumption. Output is produced using capital and labor input via a Cobb-Douglas function:

$$Y_t = K_t^{\alpha} L_t^{1-\alpha},$$

where labor input L_t grows at a constant rate n, and capital K_t is accumulated according to

$$\frac{dK_t}{dt} = sY_t - \delta K_t,$$

and s is the saving rate, and δ is the depreciation rate.

Let $y_t = \frac{Y_t}{L_t}$ and $k_t = \frac{K_t}{L_t}$ be the output per capita and the capital per capita. The household does not save when income is lower than a threshold $\bar{y} = \bar{k}^{\alpha}$. However, the household saves a constant fraction $\bar{s}$ if the income is above $\bar{y}$. Hence, the saving per capita is

$$sy_t = \begin{cases} 0 & \text{if } y_t < \bar{y} \\ \bar{s}(y_t - \bar{y}) & \text{if } y_t \geq \bar{y} \end{cases}$$

1. (5 points) Graph the saving rate against y_t.
2. (5 points) Write down the dynamic equation of capital per capita. Denote the changes in capital per capita as $\frac{dk_t}{dt}$.
3. (5 points) Write down the steady state condition.
4. (5 points) Plot a diagram depicting sy_t and $(n+\delta)k_t$. For comparison also plot the sy_t without subsistence ($\bar{y} = 0$).
5. (5 points) To ensure there exists a steady state, suppose that $\bar{y}$ is not too large. Discuss how many steady states this model may have. Plot the corresponding diagram for each possible case.
6. (5 points) Based on the cases you discussed in Question 5, characterize which steady state the economy may converge to given different values of initial capital.

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Problem 3 (Money in the Utility)

Consider a 2-period model in which both consumption c and money holding m^d provide utility directly. The representative household's utility function is given by:

$$U\left(c_t, c_{t+1}, \frac{m_t^d}{p_t}\right) = \ln c_t + \ln\left(\frac{m_t^d}{p_t}\right) + \beta \ln c_{t+1},$$

where β is the discount factor, and p_t denotes the money price of consumption goods which is taken as given. With the exogenous real income y_t and y_{t+1}, the household pays tax T_t and T_{t+1} and consumes goods and holds money. Assume $y_t > T_t$ for all period t. Meanwhile, the household also saves in government bonds b_t^d and earns nominal return on bonds i_t.

Thus the household's first-period budget constraint in nominal terms is:

$$p_t c_t + p_t b_t^d + m_t^d = p_t y_t - p_t T_t,$$

and the second-period budget constraint in nominal terms is:

$$p_{t+1} c_{t+1} = p_{t+1} y_{t+1} + (1 + i_t) p_t b_t^d + m_t^d - p_{t+1} T_{t+1}.$$

Government raises revenue via taxes and issue government bonds and money to support expenditure G. Hence, government's budget constraints in period t and $t+1$ are:

$$p_t G_t = p_t T_t + p_t b_t^s + m_t^s$$

$$p_{t+1} G_{t+1} + (1 + i_t) p_t b_t^s = p_{t+1} T_{t+1}$$

1. (5 points) Using the Fisher equation to write down the real intertemporal budget constraint.

2. (5 points) Derive the first order conditions of the consumer's utility maximization problem.

3. (5 points) Find the money demand function
$$\frac{m_t^d}{p_t} = m(i_t, y_t, \ldots).$$

4. (5 points) Show that
$$\frac{\partial m(i_t, y_t, \ldots)}{\partial i_t} < 0.$$

5. (5 points) Show that
$$\frac{\partial m(i_t, y_t, \ldots)}{\partial y_t} > 0.$$

6. (5 points) Define the competitive equilibrium for this economy.

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Problem 4 (Search)

Consider a search model of the labor market. All jobs on the market are the same and pay a wage of w units of consumption. Unemployed workers get $b < w$ in unemployment benefit, and search a job with the job finding rate being 50%. Employed workers earn wage w, but may get fired at a separation rate of 50%. Workers have linear preference over consumption: $U(C) = C$, and discount the future with factor $\beta = 1/2$.

1. (5 points) The equilibrium unemployment rate, denoted as u, occurs when flows into and out of unemployment are equal. Find the equilibrium unemployment rate u.

2. (5 points) Denote V_e and V_u as the life-time value (i.e., the total expected discounted utility) of an employed worker and an unemployed worker, respectively. Write down the relationship between V_e and V_u. Find the values V_e and V_u.

   Hint: Suppose the life-time value function is V, then it can be expressed recursively as:

$$V = U(C) + \beta E[V'],$$

   where V and $V' \in \{V_e, V_u\}$.