

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (30%) Consider a group of consumers whose preference can be represented by $u(x, y) = ax + y$ where x and y each denotes the quantity consumed of good X and Y with $a > 0$ as a preference parameter. Suppose the income of each consumer is normalized to \$1, and the size of population is normalized to 1 as well.
 - a. (5%) Suppose good X and Y are sold in competitive markets with the prices p_x and p_y respectively, derive the demand functions for the two goods.
 - b. (10%) Suppose the demand function for good X derived in the previous question is positive. If the market for good X is competitive and firms produce with the cost functions $C_x(x) = b + dx^2$ where $b, d > 0$. Derive its long-run equilibrium price.
 - c. (10%) Suppose good Y can also be produced by any interested firm with cost functions $C_y(y) = e + fy^2$ with $b < e$ and $d > f$, derive the condition for good Y, instead of X, to be the only good sold in the market.
 - d. (5%) Continue on question c, how many firms will survive in the long-run equilibrium with only good Y to be sold in the market?

2. (25%) Consider a monopolist facing the market demand $Q = 100 - P$ while producing with the cost function $C(Q) = 20Q + 1,200$.
 - a. (5%) Find the monopolist's maximized profit.
 - b. (5%) Derive the fair-return regulating price so that the industry is as efficient as possible provided that the monopolist is not suffering a loss.
 - c. (5%) Find the deadweight loss given the policy in question b.
 - d. (10%) Suppose the production of the monopoly causes total external costs of $C(Q) = Q^2$ to the society when and Q units are produced. Instead of the price regulation in question c, if the government will correct with the Pigouvian (progressive) tax, find the tax revenue it will collect.

3. (15%) Consider a duopoly industry where Firm 1 and 2 respectively produce with cost function $C_1(q_1) = 400 + 2q_1^2$ and $C_2(q_2) = 800 + q_2^2$. Suppose they engage in quantity competition with the inverse market demand as $P = 220 - 2Q$ when Q is the total output level produced collectively by the 2 firms.
 - a. (10%) Find the equilibrium price in this market.
 - b. (5%) Suppose the government imposes a tax of \$11 per unit for goods sold by the two firms, find again the equilibrium output of Firm 1.

4. (15%) Suppose Alex's utility function is $U(C, F) = 2C^{1/2} + F$ and his income is M while prices are p_C for Cloth (C) and p_F for Food (F) per unit. Assume $M > \frac{p_F^2}{p_C}$.
- (5%) Derive his demand functions for the two goods.
 - (5%) Derive his price consumption curve when p_C varies.
 - (5%) Derive the slope of his price consumption curve when p_C varies. Discuss its implications.
5. (15%) Suppose the market demand in a perfectly competitive market is $Q = 400 - 10P$ while its market supply is $Q = 10P$.
- (5%) When the government imposes a price ceiling of \$10, suppose the goods were acquired only by those who're willing to buy with the higher prices. Evaluate the deadweight loss of the price ceiling.
 - (5%) When the government imposes a price ceiling of \$10, suppose the goods were acquired randomly by those who're willing to buy at the price. This possible lost in efficiency is called allocative cost as in Davis and Kilian (2011). Evaluate the deadweight loss under the scenario.
 - (5%) When the government imposes a price ceiling of \$10, suppose instead the goods were acquired only by those who're willing to buy but with the lower willingness to pay. Evaluate again the deadweight loss under the scenario.