國立成功大學

110學年度碩士班招生考試試題

編　號：346

系　所：經濟學系

科　目：統計學

日　期：0202

節　次：第 1 節

備　註：不可使用計算機

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

※ **Instruction: No point will be given if <u>there is no explanation, no calculation, or your answers aren't understandable.</u>**

1. [25 pts] Suppose the police have caught a suspect who is charged with a felony. Assume that seventy-five percent of the suspect are actually guilty. There is a 70 percent chance the police will convict the person if the suspect is guilty. There is a 40 percent chance the police will convict the person if the suspect is not guilty. Assume that the police can only choose two options: to convict or to release the suspect.

(a) [10 pts] Find out the fraction of suspects who will be convicted by the police.

(b) [5 pts] Given that a suspect is convicted, find out the probability the person is, in fact, guilty.

(c) [5 pts] Find out the probability that the police will make a correct decision.

(d) [5 pts] Given that the police make an incorrect decision, find out the probability that the decision is to release a guilty person.

2. [40 pts] Solve the following questions:

(a) [20 pts] Let $X \sim \text{Binomial}(n, p)$, $Y \sim \text{Binomial}(m, p)$ and independent of X. Can $X - Y$ be Binomially distributed? Why?

(b) [20 pts] Show that $\text{Var}(Y) = E\big(Var(Y|X)\big) + Var(E(Y|X))$

3. [35 pts] Susan has four dice: one 4-sided die, one 6-sided die, and two 8-sided dice. Susan secretly grabs one of the four dice at random. Let S be the number of sides on the chosen die.

(a) [15 pts] Write down the probability mass function of S.

Now Susan rolls the chosen die but without showing it to you. Let R be the result of the roll.

(b) [15 pts] Find $P(S = k|R = 3)$ for $k = 4,\ 6,\ 8$. If $R = 3$, which die is most likely to be rolled by Susan?

(c) [5 pts] If $R = 7$, which die is most likely to be rolled by Susan?