

國立成功大學

111學年度碩士班招生考試試題

編 號：338

系 所：經濟學系

科 目：統計學

日 期：0219

節 次：第 1 節

備 註：不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

No credit is given to any answer without correct reasoning.

1. (15 pts) Suppose that random variables X_1 and X_2 have a continuous joint distribution for which the joint p.d.f. is as follows

$$f(x_1, x_2) = \begin{cases} 4x_1x_2, & \text{for } 0 < x_1 < 1 \text{ and } 0 < x_2 < 1 \\ 0, & \text{otherwise.} \end{cases}$$

- (a) (10 pts) Find the joint p.d.f of $Y_1 = X_1/X_2$ and $Y_2 = X_1$.
- (b) (5 pts) Find the marginal p.d.f. of Y_1 .
2. (10 pts) Given that $\{X_i\}_{i=1}^n \sim i.i.d. N(\mu, \sigma^2)$, where mean μ and variance σ^2 are both unknown. Find a $(1 - \alpha)100\%$ confidence region for (μ, σ^2) .
3. (45 pts) Given that $\{X_i\}_{i=1}^n \sim i.i.d. N(0, \sigma^2)$, where variance σ^2 is unknown. Answer the following questions.
- (a) (5 pts) Find the method of moments estimator of σ^2 and denote it as $\hat{\sigma}^2$.
- (b) (5 pts) It can be shown that the maximum likelihood estimator and method of moments estimator are the same in this case. Therefore, the realization of $\hat{\sigma}^2$ is the value of the parameter that appears to be the most likely given the data $\{X_i = x_i\}_{i=1}^n$. True or false? Explain.
- (c) (5 pts) Is $\hat{\sigma}^2$ a consistent estimator of σ^2 ?
- (d) (10 pts) Use $\hat{\sigma}^2$ to construct an exact $(1 - \alpha)100\%$ interval estimator for σ^2 .
- (e) (10 pts) Use $\hat{\sigma}^2$ to construct an approximate $(1 - \alpha)100\%$ interval estimator for σ^2 .
- (f) (10 pts) Find the asymptotic distribution of $1/\hat{\sigma}^2$.

4. (10 pts) Suppose that a sequence of Bernoulli trials is to be carried out with unknown probability p of success on each trial, and the following hypotheses are to be tested:

$$H_0 : p = p_0$$

$$H_1 : p > p_0.$$

Let $p_1 > p_0$ be a possible value of the parameter p that is consistent with the alternative hypothesis. To make the probabilities of Type I and Type II errors under our hypothesis test equal α and β , what sample size n and critical value c should we choose?

5. (20 pts) Suppose we want to estimate the model $E(Y|X) = \alpha + \beta X$. Using data $\{Y_i, X_i\}_{i=1}^n$, we regress Y on X to get $\hat{Y} = \hat{\alpha} + \hat{\beta}X$. Evaluate whether the following statements are true or false with brief explanation.
- (a) (5 pts) Economists are usually more interested in the slope coefficient β because $\hat{\beta}$ is an unbiased and consistent estimator of β .
 - (b) (5 pts) $\sum_{i=1}^n (Y_i - \alpha - \beta X_i)^2 < \sum_{i=1}^n (Y_i - \hat{\alpha} - \hat{\beta} X_i)^2$ since α and β are population parameters.
 - (c) (5 pts) R^2 from this regression can be used to determine whether X causes Y .
 - (d) (5 pts) $\sum_{i=1}^n \hat{Y}_i (Y_i - \hat{Y}_i) = 0$.