本試題是否可以使用計算機： ☑可使用 ， ☐不可使用 （請命題老師勾選）

Answer all FIVE questions, which are worth 100 points, in the paper provided and show your work. The distribution table is attached to the end of the exam. Remember, you get full credit only if you justify your steps and just correct answer does not ensure you full credit (unless specified otherwise).

1. Explain carefully whether the following functions are the probability density functions (*No points will be awarded without any justification.*):

   (a) (5 points)

$$f_1(a) = \begin{cases} 3, & 0 \le a < 1/2, \\ -1, & 1/2 \le a \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

   (b) (5 points)

$$f_2(a) = \begin{cases} (a+1)/2, & 0 \le a \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

   (c) (5 points)

$$f_3(a) = \begin{cases} 2-4a, & 0 \le a < 1/2, \\ 4a-2, & 1/2 \le a \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

2. (20 points) Assume the rise or the drop of the daily stock price is an independent and identically distributed (i.i.d.) random variable. Suppose that the probability of the rise in price for a company's stock is p, and its daily stock price will either increase by 1 dollar or drop by 1 dollar. Now an investor purchases this stock at 20 dollars, what is the average profit (the expected profit) that she earns by selling the stock after 5 trading days? What is the variance of the profit? If she holds the stock for 10 trading days and then sells, how is this different from the case of holding 5 days?

（背面仍有題目,請繼續作答）

3. (16 points) $\{X_1, \cdots, X_n\}$ and $\{Y_1, \cdots, Y_n\}$ are two i.i.d. random variables. Based on the principal of the analog estimation, we would use the sample average, $\overline{X}_n = (1/n)\sum_{i=1}^n X_i$, to estimate the population mean, $\boldsymbol{E}(X_i) = \mu_0$ where $i = 1, \ldots, n$. Accordingly, how would you estimate $\text{var}(X_i^2)$ and $\text{cov}(X_i^2, Y_i^2)$?

4. (23 points) In the population of $N(\mu, \sigma^2)$, we randomly draw two sets of independent samples $\{X_1, X_2, \ldots, X_n\}$ and $\{Y_1, Y_2, \ldots, Y_m\}$, two different estimators are proposed to estimate μ:

$$\widehat{\mu}_1 = \frac{\overline{X}_n + \overline{Y}_m}{2}, \qquad \widehat{\mu}_2 = \frac{n\overline{X}_n + m\overline{Y}_m}{n+m},$$

where $\overline{X}_n = (1/n)\sum_{i=1}^n X_i$ and $\overline{Y}_m = (1/m)\sum_{j=1}^m Y_j$.

Discuss carefully the unbiasedness and the consistency of these two proposed estimators, and compare their relative efficiency.

5. In the past, a manufacturer on average produces 500 widgets per hour. Now in order to enhance the production efficiency, an expert is hired to develop a computerized manufacturing process. 25 hours are independently drawn from such production process, it is found that the average amount of 580 widgets is produced per hour, with the standard deviation of 120. Use the large-sample test to answer the following questions:

   (a) (8 points) At a 5% significance level, test for the effectiveness of the computerized manufacturing process.

   (b) (8 points) Given that the true population mean of production is 540 widgets, at a 5% significance level, calculate the probability of Type II error and the power of the hypothesis testing performed in part (a).

   (c) (10 points) If we want to reduce Type II error to about 20%, then how much Type I error will have to increase?

本試題是否可以使用計算機： ☑可使用 ， □不可使用 （請命題老師勾選）

Table 1 Area Under the Standard Normal Distribution

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3079	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4773	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4983	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

Source: This table was generated using the SAS® function PROBNORM.