※ 考生請注意：本試題 ☑可 □不可 使用計算機

1. Suppose you have two independent unbiased estimators of the same parameter, θ, saying $\hat{\theta}_x$ and $\hat{\theta}_y$, with different variances, σ_x^2 and σ_y^2. What linear combination, $\hat{\theta} = c_x\hat{\theta}_x + c_y\hat{\theta}_y$ is the minimum variance unbiased estimator of θ? (10%).

2. In the model,

$$y_t = \beta x_t + u_t \quad (t=1,2)$$

the y_t are observable random variables, the x_t are known to take on the fixed values $x_1 = -1$, $x_2 = 0$ and $\beta = 4$.

Suppose that u_1 and u_2 have the following joint probability distribution:

(u_1, u_2)	Probability
(1,1)	1/10
(1,-1)	4/10
(-1,1)	4/10
(-1,-1)	1/10

2a) What is the variance-covariance matrix of the disturbances? Are the disturbances heteroskedastic? Are they corrected? (10%)

2b) Please find the sampling distributions of the OLS and GLS (generalized least squares) estimators of β. (15%)

2c) Please verify that $Var(\hat{\beta}_{OLS}) > Var(\tilde{\beta}_{GLS})$. (10%)

3. Consider an AR(1) regression model as the following,

$y_t = \eta y_{t-1} + u_t$

where u_t is i.i.d. $(0, \sigma^2)$

Is $\hat{\eta}$ an unbiased estimator for η? (10%)

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4. Indicate whether each statement is true or false, and give a brief explanation.

4a) As compared with unconstrained regression, estimation of a least squares regression under a constraint (saying, $\beta_2 = \beta_3$) will result in a higher R^2 if the constraint is true and a lower R^2 if it is false. (6%)

4b). If the matrix X is stochastic, least squares coefficients are unbiased but inefficient. In addition, the usual t- and F-tests involving least squares estimators are no longer valid. (6%)

4c) I am fitting a demand for car function for a sample of 10,000 housholds. I obtain an R^2 of only 0.02, but the regression program indicates that the F-statistic for the equation is very significant and so are the t-spastics. How can it be? Is there a mistake in the program? (6%)

5. We know that y_{1t} i.i.d. $(0, \sigma_1^2)$, y_{2t} i.i.d. $(0, \sigma_2^2)$, and y_{1t} and y_{2t} are independent with each other.

Suppose you want to estimate the following regression model by OLS
$y_{1t} = (y_{1t} - y_{2t})\beta + u_t$

If you want to test $H_0 : \beta = 0$ based on OLS estimator of β, Does this hull hypothesis also indicate it tests some information about σ_1^2 or σ_2^2? If it does, what information is it ? (20%)

6. Please derive the Moment Generating Function (MGF) of the Geometric Distribution. (7%)