※ 考生請注意：本試題 ☑可 ☐不可 使用計算機

Answer all FIVE questions, which are worth 100 points, in the paper provided and show your work. The distribution table is attached to the end of the exam. Remember, you get full credit only if you justify your steps and just correct answer does not ensure you full credit (unless specified otherwise).

1. Let $\overline{Y}$ denote the sample average from a random sample with mean μ and variance σ^2. Consider two alternative estimators of μ: $W_1 = [(n-1)/n]\overline{Y}$ and $W_2 = \overline{Y}/2$.

   (a) (10 points) Show that W_1 and W_2 are both biased estimators of μ and find the biases. What happens to the biases as $n \to \infty$? Comment on any important differences in bias for the two estimators as the sample size gets large.

   (b) (10 points) Find the probability limits of W_1 and W_2. Which estimator is consistent?

   (c) (8 points) Find $\text{Var}(W_1)$ and $\text{Var}(W_2)$

   (d) (5 points) Argue that W_1 is a better estimator than $\overline{Y}$ if μ is "close" to zero. (Consider both bias and variance.)

2. You are hired by the mayor to study whether a tax on liquor has decreased average liquor consumption in your city. You are able to obtain, for a sample of individuals selected at random, the difference in liquor consumption (in ounces) for the years before and after the tax. For person i who is sampled randomly from the population, Y_i denotes the change in liquor consumption. Treat these as a random sample from a Normal(μ, σ^2) distribution. Suppose your sample size is $n = 900$ and you obtain the estimates of sample average and sample standard deviation, $\overline{y} = -32.8$ and $s = 466.4$, respectively.

   (a) (10 points) Formally test the hypothesis that a tax on liquor has decreased average liquor consumption in your city at the 5% level of significance. (i) State the null and alternative hypotheses; (ii) calculate the test statistic; (iii) Obtain the p-value for the test; (iv) State your conclusion.

(背面仍有題目,請繼續作答)

※ 考生請注意：本試題 ☑可 ☐不可 使用計算機

(b) (8 points) Explain the differences and similarities between the "level of significance" and the "level of confidence."

(c) (6 points) The results in (a) show that we are 95% confident that μ is negative. True or false? If false, explain.

3. (15 points) Given that the cumulative distribution function of X is $F_X(a) = a^2/36$. Calculate the probability density function, mean and variance of X

4. (16 points) Suppose that X and Y are random variables. The mean of X is μ_X and the variance is 4, while the mean of Y is 0 and the variance is σ_Y^2. Suppose that we know $\mathbb{E}(X|Y) = 2Y^2$ $\mathbb{E}(Y|X) = -3 + 0.5X$ Calculate μ_X, σ_Y^2 and $\text{cov}(X,Y)$. If we also know $\text{var}(Y|X)$ is a constant, c, calculate $\text{var}(Y|X)$.

5. (12 points) $\{(X_1, Y_1), \quad , (X_n, Y_n)\}$ is a sequence of *i.i.d.* random variables, and $\mathbb{E}(Y_i|X_i) = 2X_i$. The linear regression model under consideration is

$$Y_i = \alpha + \beta X_i + U_i, \qquad i = 1. \quad , n.$$

Is the least squares estimator of β a consistent estimator of the parameter, 2? Explain.

※ 考生請注意：本試題 ☑可 □不可 使用計算機

Table 1 Area Under the Standard Normal Distribution

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3079	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4773	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4983	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

Source: This table was generated using the SAS® function PROBNORM.