

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

**A. (10% with 5% each)**

Prove that if A and B are independent events,

1.  $\bar{A}$  and B are independent.
2.  $\bar{A}$  and  $\bar{B}$  are independent.

**B. (10%)**

Suppose that A and B are independent events associated with an experiment. If the probability that A or B occurs equals 0.6. The probability that A occurs equals 0.4. What is the probability that B occurs.

**C. (10%)**

We have two urns, 1 and 2, each with two drawers. Urn 1 has a gold coin in one drawer and a silver coin in the other drawer. Urn 2 has a gold coin in each coin. One urn is chosen at random; then a drawer is chosen at random from the chosen urn. The coin found in this drawer is a gold coin. What is the probability that the coin came from urn 2?

**D. (10%)**

If the random variable K is uniformly distributed over (0, 5), what is the probability that the roots of the equation  $4x^2 + 4xK + K + 2 = 0$  are real?

**E. (10% with 5% each)**

Assume that  $E(X_1) = E(X_2) = 1.5$ ,  $\text{Var}(X_1) = \text{Var}(X_2) = 0.25$ , and the correlation coefficient between  $X_1$  and  $X_2$  is 0.5. Let  $D = 3X_1 - 2X_2$ ,

1. The expected value of D?
2. The variance of D?

**F. (15% with 5% each)**

Suppose that the two-dimensional random variable (X, Y) has joint pdf

$f(x, y) = kx(x - y)$  for  $0 < x < 2$ ,  $-x < y < x$ ;  $f(x, y) = 0$ , elsewhere.

1. Evaluate the constant k.
2. Find the marginal pdf of X.
3. Find the marginal pdf of Y.

**G. (20% with 10% each)**

Suppose that the joint pdf of the two-dimensional random variable  $(X, Y)$  is given by  $f(x, y) = x^2 + \frac{xy}{3}$  for  $0 < x < 1, 0 < y < 2$ ;  $f(x, y) = 0$ , elsewhere. Compute the following.

1.  $\Pr(X > 0.5)$ ;
2.  $\Pr(Y < X)$ .

**H. (15% with 5% each)**

Assume that birthweights are normally distributed with a mean of 3400 g and a standard deviation of 700 g.

1. Find the probability of a low-birthweight child, where low birthweight is defined as  $\leq 2500$  g.
2. Find the probability of a very low birthweight child, where very low birthweight is defined as  $\leq 2000$  g.
3. Assuming that successive deliveries by the same woman have the same probability of being low birthweight, what is the probability that a woman with exactly 3 deliveries will have 2 or more low birthweight deliveries?