

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (18%) Evaluate the following limits:

$$(a) \lim_{n \rightarrow \infty} \frac{\sqrt{1} + \sqrt{2} + \dots + \sqrt{n}}{n^{3/2}} \quad (b) \lim_{x \rightarrow \frac{\pi}{4}} (\tan x)^{\tan 2x} \quad (c) \lim_{x \rightarrow -\infty} \frac{1+x}{5x-6}$$

2. (12%) Let  $f(x)$  be a continuous function, and

$$\int_0^x f(u) du = -2 + x^2 + x \sin 2x + c \cos 2x.$$

(a) Find the value of  $c$ .

(b) Evaluate  $\int_{\pi/4}^{\pi/2} f(x) dx$ .

(c) Find  $f'(\frac{\pi}{4})$ .

3. (10%) Find the Maclaurin series for  $f(x) = \ln(1 + 2x)$  and find the interval of convergence and the radius of convergence of this series.

4. (18%) Evaluate the following integrals:

$$(a) \int_{-1}^1 \frac{1}{x^2} dx \quad (b) \int \frac{1}{\sqrt{x+\sqrt{x}}} dx \quad (c) \int_{-4}^3 |x^2 - x - 6| dx$$

5. (10%) Test  $f(x, y) = x^3 + y^3 + xy - 4$  for maxima, minima, or saddle points.

6. (10%) Let  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  be a continuously differentiable function and  $f(1, 1) = 1$ ,  $\frac{\partial f}{\partial x}(1, 1) = a$ ,  $\frac{\partial f}{\partial y}(1, 1) = b$ . If  $g(x) = f(x, x)$  and  $h(x) = f(x, f(x, x))$ , find (a)  $g'(1)$  (b)  $h'(1)$ .

7. (12%) Let  $u(x, y) = x^3 + y^3 - 3xy$ , where  $x = \tan^{-1}(r^2 + \sin s)$  and  $y = (5r)^{\sin s}$ . Find  $\frac{\partial u}{\partial r}$  and  $\frac{\partial u}{\partial s}$ .

8. (10%) Evaluate the double integral  $\int_0^1 \int_y^1 e^{x^2} dx dy$ .