

※ 考生請注意：本試題可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

A. (20% with 10% each)

1. Suppose we have a sample of 100 men, of whom 30 have breast cancer and 70 do not. If the prevalence of breast cancer = p , then what is the likelihood of the sample given p ?
2. Suppose we have n independent observations x_1, \dots, x_n from a normal distribution with mean = μ and variance = σ^2 . What is the likelihood of the sample?

B. (25% with 10% for #1, #3 each and 5% for #2)

Suppose a disease is caused by a single major gene with two alleles (a) and (A) with frequencies .95 and .05, respectively.

1. What is the probability that an individual will have genotype (aa), (aA), and (AA), if we assume that each allele is inherited independently? (That is, obtain the probabilities of $P(aa)$, $P(aA)$, and $P(AA)$.)
2. Suppose the A allele is the deleterious allele but that the gene is only partially penetrant, meaning that the probability of developing the disease is .9 if one has two A alleles, .5 if one has one A allele, and .1 if one has no A alleles (sporadic cases). What is the overall probability of developing the disease in the population? (That is, obtain the probability of $P(\text{Disease})$.)
3. Suppose an individual has the disease. What is the probability that he or she will have no, one, or two A alleles? (That is, obtain the probabilities of $P(aa | \text{Disease})$, $P(aA | \text{Disease})$, and $P(AA | \text{Disease})$.)

C. (10% with 5% each)

Assume that $E(X_1) = E(X_2) = 1.5$, $\text{Var}(X_1) = \text{Var}(X_2) = 0.25$, and the correlation coefficient between X_1 and X_2 is 0.5. Let $D = X_1 - X_2$,

1. The expected value of D?
2. The variance of D?

D. (15% with 5% each)

A recent study of incidence rates of blindness among insulin-dependent diabetics reported that the annual incidence rate of blindness per year was 0.67% among 30- to 39-year-old male insulin-dependent diabetics (IDDM) and 0.74% among 30- to 39-year-old female insulin-dependent diabetics.

1. If a group of 200 IDDM 30- to 39-year-old men is followed, what is the probability that exactly 2 will go blind over a 1-year period?
2. If a group of 200 IDDM 30- to 39-year-old women is followed, what is the probability that at least 2 will go blind over a 1-year period?
3. What is the probability that a 30-year-old IDDM male patient will go blind over the next 10 years?

E. (15% with 5% each)

Suppose the number of people seen for violent asthma attacks in the emergency ward of a hospital over a 1-day period is usually Poisson distributed with parameter $\lambda = 1.5$.

1. What is the probability of observing 5 or more cases over a 2-day period?
2. On a particular 2-day period, the air-pollution levels increase dramatically and the distribution of attacks over a 1-day period is now estimated to be Poisson distributed with parameter $\lambda = 3$.
3. What is the probability of observing 5 or more cases over a 2-day period?
3. If 10 days out of every year are high-pollution days, then what is the expected number of asthma cases seen in the emergency ward over a 1-year period? (Assume there are 365 days in a year.)

F. (15%)

Let X be a discrete random variable. We define the k -th factorial moment of X as $m_k = E[X(X-1)(X-2)\dots(X-k+1)]$ where k is a positive integer. Calculate m_k assuming that X is a Poisson distribution with parameter μ .

