

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (12%) Evaluate the following limits:

(a) Suppose  $\lim_{x \rightarrow \infty} f'(x) = A$ ,  $a \neq 0$ , find  $\lim_{x \rightarrow \infty} \{f(x+a) - f(x)\} = ?$ .

(b)  $\lim_{x \rightarrow \frac{\pi}{4}} \tan 2x \cdot \tan\left(\frac{\pi}{4} - x\right)$ .

2. (14%) Let  $f(x)$  be a continuous function.

(a) If  $\int_0^{f(x)} t^2 dt = x \cos \pi x$ , find  $f(9)$ .

(b) If  $\int_0^{x^2} f(t) dt = x \sin \pi x$ , find  $f(9)$ .

3. (12%) Evaluate

$$\lim_{a \rightarrow 0^+} \frac{1}{\sqrt{a}} \left( \lim_{x \rightarrow a} \frac{1}{x-a} \int_{\sin a}^{\sin x} \sqrt[4]{t^2 + a^2} dt \right).$$

4. (14%) Find the Maclaurin series for  $f(x) = \ln(1+2x)$  and find the interval of convergence and the radius of convergence of this series.

5. (12%) Evaluate the following integrals:

(a)  $\int_{-1}^1 \frac{1}{x^2} dx$ .

(b)  $\int_5^{\infty} \left( \ln \frac{x+5}{x} - \frac{5}{x+5} \right) dx$ .

6. (12%) Test  $f(x, y) = x^3 + y^3 + xy - 4$  for maxima, minia, or saddle points.

7. (10%) Let  $f: \mathbb{R}^2 \rightarrow \mathbb{R}$  be a continuously differentiable function and  $\frac{\partial f}{\partial x}(1, 2) = a$ ,  $\frac{\partial f}{\partial y}(1, 2) = b$ . If  $g(x) = f(x, 2x)$ , find  $g'(1)$ .

8. (14%) Let  $R = [-2, 1] \times [0, 2]$ . Evaluate the double integral

$$\iint_R |y - x^2| dx dy.$$