國立成功大學 106 學年度碩士班招生考試試題

系 所：環境醫學研究所

考試科目：微積分

考試日期：0214，節次：3

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

作答時請列出計算過程,若只寫答案則不予計分.

1. (20%) Evaluate the following limits:

   (a) $\lim_{x\to 0^+}(e^{3x}+2x)^{1/x}$.

   (b) If $f:\mathbb{R}\to\mathbb{R}$ is continuous at a and $\lim_{x\to 0}\frac{f(a+\tan x)}{x}=5$, find $\lim_{x\to 0}\frac{f(a+\tan x)}{\tan x}$.

   (c) $\lim_{n\to+\infty}\int_0^{2\pi}\frac{\sin nx}{x^2+n^2}\,dx$.

   (d) $\lim_{t\to 0}\frac{1}{\ln t}\int_t^1\frac{\sin x}{x}\,dx$.

2. (10%) Let $f(x)$ be a differentiable fuinction.

   (a) If $f(\frac{3x-2}{2x+1})=x$, find $f'(1)$.

   (b) If $\int_0^{x^2} f(t)\,dt = x\sin\pi x$, find $f(9)$.

3. (20 %) Sketch the graph of $f(x)=\int_0^{x^2}\sqrt{t^4+2}\,dt$, $x\in\mathbb{R}$, showing its first and second derivatives, critical points, points of inflection, the intervals on which it increases or decreases, and the intervals on which the graph is concave up or concave down.

4. (15%) Let $\Gamma(\alpha)=\int_0^\infty e^{-x}x^{\alpha-1}\,dx$.

   (i) Prove that the improper integral $\Gamma(\alpha)$ is convergent for $\alpha>0$.

   (ii) Prove that $\Gamma(n)=(n-1)!$ for any $n\in\mathbb{N}$.

5. (10%) Evaluate the iterated integral $\int_0^2\int_{2y}^4 e^{x^2}\,dxdy$.

6. (15%) Let $z=z(x,y)$ be differentiable, $x=r\cos\theta$, $y=r\sin\theta$. Prove that
$$\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=\frac{\partial^2 z}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2 z}{\partial\theta^2}.$$

7. (10%) Let $f:\mathbb{R}^2\to\mathbb{R}$ be a continuously differentiable function and $\frac{\partial f}{\partial x}(1,2)=a$, $\frac{\partial f}{\partial y}(1,2)=b$. If $g(x)=f(x,2x)$, find $g'(1)$.