

※ 考生請注意：本試題可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。作答時請列出計算過程，若只寫答案則不予計分

A. (20% with 10% each)

1. A, B, and C are three events. Prove $P(A \cap B \cap C) = P(A|B \cap C)P(B|C)P(C)$.
2. Prove that if A and B are independent events, then \bar{A} and \bar{B} are independent.

B. (10%)

Suppose that A and B are independent events associated with an experiment. If the probability that A or B occurs equals 0.6. The probability that A occurs equals 0.4. What is the probability that B occurs.

C. (20%)

A point is chosen at random on a line of length L . What is the probability that the ratio of the short segment to the longer segment is less than $\frac{1}{4}$? (Hint: Use a uniform distribution.)

D. (30% with 10% each)

Suppose that the joint pdf of the two-dimensional random variable (X, Y) is given by $f(x, y) = x^2 + \frac{xy}{3}$ for $0 < x < 1, 0 < y < 2$; $f(x, y) = 0$, elsewhere. Compute the following.

1. $P(X > \frac{1}{2})$;
2. $P(Y < X)$;
3. $P(Y < \frac{1}{2} | X < \frac{1}{2})$.

E. (20%)

Suppose that X and Y are two random variables for which $Y = AX + B$, where A and B are constants. Then $\rho^2 = 1$. If $A > 0$, $\rho = 1$; if $A < 0$, $\rho = -1$. The symbol ρ is the correlation coefficient between X and Y.