國立成功大學

110學年度碩士班招生考試試題

編　號：296

系　所：環境醫學研究所

科　目：機率與統計

日　期：0203

節　次：第 2 節

備　註：可使用計算機

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※ 考生請注意：本試題可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

Please provide complete and detailed calculations. If only answers are provided, then no credits are to be given.

A. (20% with 10% each)

1. A, B, and C are three events. Prove $P(A \cap B \cap C) = P(A|B \cap C)P(B|C)P(C)$.
2. Prove that if A and B are independent events, then $\bar{A}$ and $\bar{B}$ are independent.

B. (20%)

If the random variable K is uniformly distributed over (0, 5), what is the probability that the roots of the equation $4x^2 + 4xK + K + 2 = 0$ are real?

C. (30% with 10% each)

Suppose that the two-dimensional random variable (X, Y) has joint pdf
$f(x, y) = kx(x - y)$ for $0<x<2$, $-x<y<x$; $f(x, y) = 0$, elsewhere.

1. Evaluate the constant k.
2. Find the marginal pdf of X.
3. Find the marginal pdf of Y.

D. (20% with 10% each)

1. Suppose we have a sample of 100 women, of whom 30 have breast cancer and 70 do not. If the probability for a woman to have breast cancer is p, then what is the likelihood of the sample given p?
2. Suppose we have n independent observations $x_1, \ldots, x_n$ from a normal distribution with mean = μ and variance = σ^2. What is the likelihood of the sample?

E. (10%)

Let X be a discrete random variable. We define the k-th factorial moment of X as $m_k = E[X(X-1)(X-2)...(X-k+1)]$ where k is a positive integer. Calculate m_k assuming that X is a Poisson distribution with parameter μ.