

國立成功大學
110學年度碩士班招生考試試題

編 號：300

系 所：環境醫學研究所

科 目：微積分

日 期：0203

節 次：第 3 節

備 註：不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (20%) Evaluate the following limits:

(a) $\lim_{x \rightarrow 0^+} x^n \ln x.$

(b) $\lim_{x \rightarrow +\infty} x \ln\left(\frac{x-1}{x+1}\right).$

(c) $\lim_{x \rightarrow 0^+} x^{\tan x}.$

(d) $\lim_{x \rightarrow 0} \frac{1}{x^3} \int_0^x \frac{t^2}{t^4+1} dt.$

2. (20%) Evaluate the following integrals:

(a) $\int \frac{1}{1+\sqrt{x}} dx.$

(b) $\int_{-1}^1 \frac{1}{x^2} dx.$

(c) $\int_{\pi^2/4}^{\pi^2} \frac{\sin \sqrt{x} \cos \sqrt{x}}{\sqrt{x}} dx.$

(d) $\int_0^{\pi/2} \cos^3 x dx.$

3. (10%) Determine whether or not the improper integral $\int_{-1}^1 \frac{\tan x}{x^2} dx$ converges. Explain your reason.

4. (10%) Find the limit $\lim_{n \rightarrow \infty} \frac{\sqrt[n]{n!}}{n}$. (Hint: $\frac{\sqrt[n]{n!}}{n} = \exp\{\ln \sqrt[n]{\frac{n!}{n^n}}\}$)

5. (10%) Find the area of the region inside $r^2 = 2 \cos 2\theta$ and outside $r = 1$.

6. (10%) Suppose that $w = f(x, y)$, $x = r \cos \theta$, and $y = r \sin \theta$. Show that

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2}.$$

7. (10%) Show that $(1+x)^{3/2} > 1 + \frac{3}{2}x$ for all $x > 0$.

8. (10%) Evaluate the iterated integral $\int_0^1 \int_0^{\sqrt{1-y^2}} e^{x^2+y^2} dx dy.$