# 國立成功大學
# 111學年度碩士班招生考試試題

編　號：292

系　所：環境醫學研究所

科　目：微積分

日　期：0220

節　次：第 3 節

備　註：不可使用計算機

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考試科目：微積分

考試日期：0220，節次：3



※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。
Please provide complete and detailed calculations. If only answers are provided, then no credits are to be given.

1. (15%) Evaluate the following limits:

   (a) $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{kn^2}{(n^2+k^2)^2}$

   (b) $\lim_{x\to 0^+} x^x$

   (c) $\lim_{x\to\infty} x^2 e^{-x^3} \int_0^x e^{t^3}\, dt$

2. (10%) Let $x > -1$ and $x \neq 0$. Prove that

$$\frac{x}{1+x} < \ln(1+x) < x.$$

3. (a) (10 %) Evaluate $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}\, dx$.

   (b) (10 %) Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin^n x}{\sin^n x + \cos^n x}\, dx$, $n > 0$.

   (c) (10 %) Evaluate the iterated integral $\int_0^2 \int_{\frac{y}{2}}^1 y e^{-x^3}\, dx dy$.

4. Let $\Gamma(\alpha) = \int_0^{\infty} e^{-x} x^{\alpha-1}\, dx$.

   (a) (10 %) Prove that the improper integral $\Gamma(\alpha)$ is convergent for $\alpha > 0$.

   (b) (5 %) Prove that $\Gamma(n) = (n-1)!$ for any $n \in \mathbb{N}$.

5. (10%) Determine whether or not the improper integral $\int_1^{\infty} \tan \frac{1}{x}\, dx$ converges. Explain your reason.

6. (10%) Let $z = z(x, y)$ be twice continuously differentiable, $x = r - \theta$, $y = r + \theta$. Prove that

$$\frac{\partial^2 z}{\partial r \partial \theta} = \frac{\partial^2 f}{\partial y^2} - \frac{\partial^2 f}{\partial x^2}.$$

7. (10%) Find the absolute maximum and minimum of $f(xy) = x^2 + y^2 + y$ on the disc $x^2 + y^2 \leq 1$.