1. Find the following limits: (each 10%)

   (a) $\lim_{x \to 0}(\frac{1}{x} - \cot x)$;　　　(b) $\lim_{n \to \infty} \sum_{k=1}^{n} \frac{k}{n^2 + 2nk}$.

2. Show that $|\sin x - \cos x| \le \sqrt{2}$ for all $x \in \mathbb{R}$.　　　(10%)

3. Find the area of the largest rectangle that can be inscribed in a semicircle of radius r.　　　(10%)

4. Evaluate the following integrals: (each 10%)

   (a) $\int_{1}^{e} (x-1)\ln x\, dx$;

   (b) $\iint_{R} e^{(x^2+y^2)}\, dxdy$ where $R = \{(x,y) \mid 0 \le y \le x,\ x^2 + y^2 \le 1\}$.

5. Find the radius of convergence and interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(2x-1)^n}{n2^n}.$$

(10%)

6. Given a plane curve $\begin{cases} x = e^t + e^{-t} \\ y = 3 + 2t, \end{cases}$　$0 \le t \le 2$.

   (a) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ at $t = 1$.　　　(10%)

   (b) Find the arc length of this curve.　　　(10%)

7. Find the extreme values of the function $f(x,y) = x^3 - 3x^2y + y^2 + 2y + 5$.　　　(10%)