

系所組別：環境醫學研究所甲組

考試科目：微積分

考試日期：0307，節次：2

※ 考生請注意：本試題 可 不可 使用計算機

1. Please prove the limit theorem 4: If $\lim_{x \rightarrow a} f(x) = L$ and $\lim_{x \rightarrow a} g(x) = M$, then

$$\lim_{x \rightarrow a} [f(x) \pm g(x)] = L \pm M \quad (15\%)$$

2. If f and g are functions and if h is the function defined by $h(x) = \frac{f(x)}{g(x)}$ where $g(x) \neq 0$, then if $f'(x)$ and $g'(x)$ exist, please prove:

$$h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} \quad (15\%)$$

3. Two towns A and B are to get their water supply from the same pumping station to be located on the bank of a straight river that is 15 mile from town A and 10 mile from town B . If the points on the river nearest to A and B are 20 mile apart and A and B are on the same side of the river, where should the pumping station be located so that the least amount of piping is required?(15%)
4. Evaluate $\int \frac{dx}{x^2 \sqrt{27x^2 + 6x - 1}}$ by using the reciprocal substitution $x = 1/z$. (15%)
5. Obtain a power-series representation of $\frac{1}{(1-x)^2}$ (10%)
6. A projectile is shot from a gun at an angle of elevation of radian measure $1/6 \pi$. Its muzzle speed is 480 ft/sec. Find (a) the position vector of the projectile at any time; (b) the time of flight; (c) the range; (d) the maximum height; (e) the velocity vector of the projectile at impact; (f) the position vector and the velocity vector at 2 sec; (g) the speed at 2 sec; (h) a Cartesian equation of the curve traveled by the projectile. (20%)

7. Evaluate the double integral $\int_R \int (2x^2 - 3y) dA$

if R is the region consisting of all points (x, y) for which $-1 \leq x \leq 2$ and $1 \leq y \leq 3$.

(10%)