系所組別： 奈米科技暨微系統工程研究所

考試科目： 工程數學　　　　　　　　　　　　　　　　　　　　考試日期：0219，節次：3

※ 考生請注意：本試題 □可　☑不可 使用計算機

1.

(a) Solve the following third-order Ordinary Differential Equation system (10 %)

$$x^3 y''' - 3x^2 y'' + (6 - x^2)xy' - (6 - x^2)y = 0$$

(b) Find the general solution in terms of Bessel's function for the following equations (10 %)

$$0.5x^2 y'' + 0.5xy' + (2x^4 - 0.125)y = 0$$

2. Verify Green's theorem by the given vector $\vec{F} = f(x,y)\vec{i} + g(x,y)\vec{j} = 3y\,\vec{i} - 2xy\,\vec{j}$

along the circle C: $(x-3)^2 + (y-2)^2 = 16$

Please find (a) $\iint_R (\dfrac{\partial g(x,y)}{\partial x} - \dfrac{\partial f(x,y)}{\partial y})\,dxdy = ?$ (10 %) , and (b) $\oint_C \vec{F} \cdot d\vec{r} = ?$ (10 %)

3. Solve the following initial boundary valued problem of $u(x,t)$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \quad , \quad u(0,t) = t \;\; ; \;\; u(1,t) = 1 \;\; ; \;\; t > 0$$

$$u(x,0) = x \;\; ; \;\; 0 < x < 1 \qquad \text{(20 %)}$$

4. Solve the solution of the following wave equation:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \quad , \text{with the following initial conditions: } u(x,0) = f(x) \;\; , \;\; \frac{\partial u(x,0)}{\partial t} = g(x)$$

And boundary conditions: $u(0,t) = 0 \;\; , \;\; u(L,t) = a\sin(wt)$

, where c , a , and ω are all constants. (20 %)

5. Prove the integral as following:

(a) $\displaystyle\int_{-\infty}^{\infty} e^{-ax^2}\,dx = \sqrt{\frac{\pi}{a}} \quad , \quad a > 0$ (10 %)

(b) $\displaystyle\int_0^{\infty} \frac{x^{p-1}}{1+x}\,dx = \frac{\pi}{\sin p\pi} \quad , \quad 0 < p < 1$ (10 %)