編號：G 211　系所：奈米科技暨微系統工程研究所　　科目：工程數學

本試題是否可以使用計算機：☑可使用，□不可使用　（請命題老師勾選）

1. Given an equation as $\frac{dx}{dt} + 2x - \int_0^t 3\,xdt = t$. If the initial condition x(0)=1, find the solution x(t) for the equation. (10%)

2. A semi-infinite string is initially at rest in a position coinciding with the positive half of the x-axis. Its left-hand end is fixed. A transverse force of magnitude F_o moves along the string a constant velocity v, beginning at t=0 at the point x=0. In this problem, the governing equation is non-homogeneous wave equation.

$$\frac{\partial^2 y}{dt^2} = a^2 \frac{\partial^2 y}{dx^2} - \frac{g}{w} F_o \delta\left(t - \frac{x}{v}\right)$$

where a, g and w are constants corresponding to the propagation speed for the string, gravity and the weight of the string per unit length. $\delta\left(t - \frac{x}{v}\right)$ is the unit impulse, or δ function.

The boundary conditions are:

$$y(0,t) = 0 \text{ and } y(x,t) \text{ bounded} \quad as \quad x \to \infty$$

and the initial conditions are:

$$y(x,0) = 0 \text{ and } \left.\frac{\partial y}{\partial t}\right|_{x,0} = 0$$

(a) Find the displacement y(x,t) of the string at any point and time when v≠a. (10%)

(b) Find the displacement y(x,t) of the string at any point and time when v=a. (10%)

(c) Plot y(x,t) of (b) for t=1 and $\frac{gF_o}{2wa} = 1$. Describe the physical meaning of the plot. (10%)

3. Please evaluate the following definite integral:

(a) $\int_0^\infty \frac{dx}{x^6 + 1}$ (10%)

(b) $\int_0^\infty \sin x^2 dx + \int_0^\infty \cos x^2 dx$ (10%)

4. (a) At what point or points is the tangent to the curve $x=t^3$, $y=5t^2$, $z=10t$ perpendicular to the tangent at the point where t=1? (10%)

(b) Find the unit vector normal to the surface $xy^3z^2=4$ at the point (-1, -1, 2) (10%)

5. (a) Find the characteristic values and corresponding characteristic vectors for matrix $A=\begin{bmatrix} 4 & 3 \\ 5 & 6 \end{bmatrix}$ (10%)

(b) Find the matrix X solutions of the equation $X^2 - 4X + 4I = A$ where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (10%)