

本試題是否可以使用計算機：可使用，不可使用（請命題老師勾選）

1. (1) Explain the meaning of gradient $\nabla\Phi$ (i.e., $\nabla\Phi$), where Φ is a scalar field. (10%)
(2) Given $\Phi = xyz^2 + x^2y^3$, find the directional derivative $d\Phi/dl$ in the direction $1\mathbf{a}_x + 2\mathbf{a}_y + 2\mathbf{a}_z$ at point (1,1,2). (10%)
2. For a coaxial cable, inner conductor radius a , outer conductor radius b ($b > a$), cable length L , the homogeneous dielectric with permittivity ϵ and conductivity σ is filled between two conductors. Assume that inner and outer conductors carry $+Q$ and $-Q$, respectively. Find (1) the capacitance of the coaxial cable and (2) the leakage resistance between two conductors. (20%)
3. A circular loop located at $x^2 + y^2 = 16$, $z = 0$ carries 5 Ampere current along \mathbf{a}_ϕ . Determine magnetic field intensity \mathbf{H} at points (0, 0, 3) and (0, 0, -3). Using cylindrical coordinates (ρ, ϕ, z) to solve it. (20%)
4. Find the Laplacian of the scalar fields shown below. (20%)
(1) $V = e^{-x} \sin(y) \sinh(z)$.
(2) $W = 5r^2 \sin(\theta) \cos^2(\phi)$.
By definition, spherical coordinates (r, θ, ϕ) and cylindrical coordinates (ρ, ϕ, z).
5. A 2-meter-long transmission line operates at $\omega = 10^6$ rad/s with $\alpha = 1$ rad/m and $\beta = 2$ rad/s, $Z_0 = 20 + j10 \Omega$. The line is connected to an input source of $2 \angle 30^\circ$ V with source resistance 40Ω and terminated by a load of $10 + j20 \Omega$. Find (1) the input impedance, (2) the sending-end current, (3) the current at one fourth of the line. (20%)