編號： 212　系所：奈米科技暨微系統工程研究所　　科目：工程數學

本試題是否可以使用計算機：☑可使用 ，☐不可使用 （請命題老師勾選）

1. **Solve the initial value problem**

$$y'' + 4y = f(t); \quad y(0)=0,\ y'(0)=1$$

when f(t) function is defined as:

(a) $f(t) = \begin{cases} t, & \text{for } 0 \le t < 1, \\ 0, & \text{for } t \ge 1, \end{cases}$ **(10%)**

(b) $f(t) = t$ for $0 \le t < 1$ and $f(t+1) = f(t)$ for $t \ge 0$ **(10%)**

2. **Solve one dimensional diffusion equation** $\dfrac{\partial^2 u}{\partial x^2} = \dfrac{1}{\alpha^2}\dfrac{\partial u}{\partial t}$

with the initial condition $u(x,0) = f(x)$ for $0<x<L$ and **the boundary conditions are defined as:**

(a) $u(0,t) = 0$; $u(L,t) = 1$; **(10%)**

(b $u(0,t) = 0$; and $\dfrac{\partial u}{\partial x}(L,t) = -hu(L,t)$,where h is a constant **(10%)**

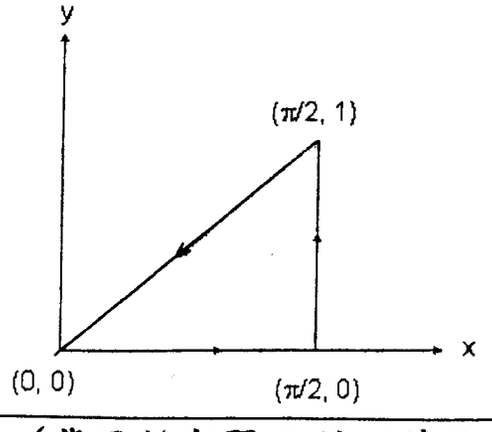
3. **Matrix** $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, solve $\mathbf{A}^{256}$? **(10%)**

4. **Solve Inverse Laplace transform** $L^{-1}\left[\dfrac{1}{\sqrt{s+1}}\right]$ **(15%)**

5. **For the close loop line integral** $\oint [(y - \sin x)dx + \cos x dy]$ **where C is the triangular in xy plane in the following figure.**

(a) Please evaluate the line integral directly. **(10%)**

(b) Please evaluate the line integral by using Green Theorem. **(10%)**



（背面仍有題目，請繼續作答）

6. Find the solution for a complex function Φ satisfies the Laplace equation:

$$\nabla^2\Phi = \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} = 0$$

The corresponding boundary conditions are:

$$\Phi = 1 \quad \text{for } y = 0,\ 0 < x < c;$$

$$\frac{\partial\Phi}{\partial y} = 0 \quad \text{for } y = 0,\ c < x < \pi;$$

$$\frac{\partial\Phi}{\partial x} = 0 \quad \text{for } x = 0; \qquad \textbf{(15\%)}$$

$$\Phi = 0 \quad \text{for } x = \pi$$

$$\frac{\partial\Phi}{\partial y} = 0 \quad \text{for } y \to \infty;$$