1. Compressibility E of a fluid is defined as $E = \dfrac{-\Delta P}{(\Delta V)/V}$, the change in volume ($V$) of a fluid that is subjected to a change in pressure. Figure 1 shows a hydraulic linear actuator with inside diameter D and length L. A measure of **stiffness** of a linear actuator is the amount of the force required to cause a certain linear deflection. For two actuators that have inside diameters of $D=D_1$ and $D=D_2$ and lengths of $L=L_1$ and $L=L_2$ and that are filled with water.

(a) Computer the stiffness ratio if $D_1/D_2=2$ and $L_1/L_2=2$. (10 %)

(c) Describe how to design the stiffness of hydraulic linear actuator by varying diameter D and length L? (10 %)
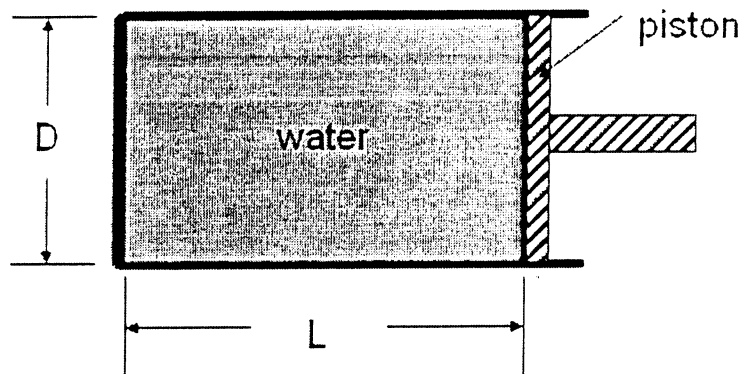

Figure 1

2. Figure 2 shows a tank (diameter D) with a smooth, well-rounded nozzle (diameter d) in the bottom through which fluid is discharging. Derive the equation for the time required to empty a tank, considering the variation of velocity as the depth decreases. (10%)
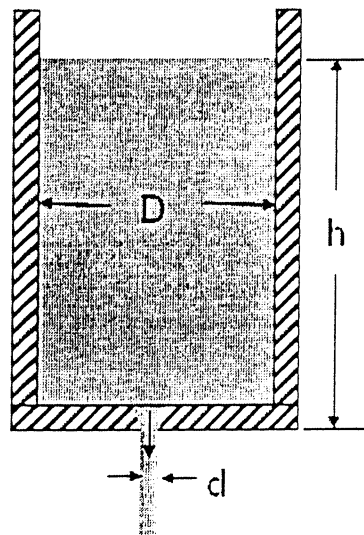

Figure 2

3. Consider a two-dimensional flow field which is formed by a source of strength Λ located above a floor. The source is located at $(0, 1)$, as shown in Figure 3.
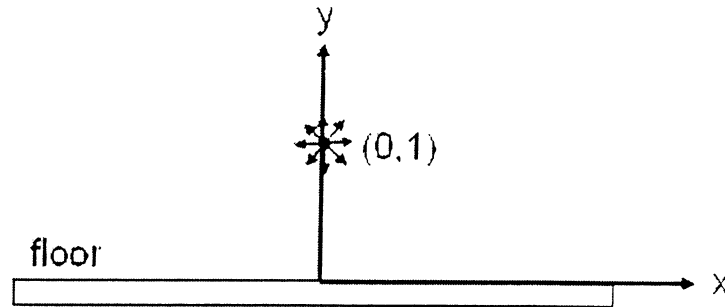

Figure 3

(i)　　Determine velocity potential ϕ for this flow field.　(10%)

(ii)　　Locate the stagnation point in this flow field.　(5%)

(iii)　　Determine the streamline equation passing through the stagnation point in the Cartesian coordinate.　(5%)

(iv)　　Where does the maximum speed along the floor occur?　(10%)

(v)　　Assume the total (or stagnation) pressure in the flow field is P_o and the density of the fluid is ρ, find the static pressure P at the point $(1,0)$.　(10%)

Hint: If a source flow located at the origin $(0, 0)$, the velocities in (r,θ) direction of polar coordinate are

$$u_r = \frac{\Lambda}{2\pi r}, \ u_\theta = 0 \ \text{where} \ r = \sqrt{x^2 + y^2}$$

4. As shown in Figure 4, the wind stress exerts a stress τ_w at the free surface which induces a Couette type of flow to the right. However, in order to conserve mass within the closed region, a return flow is required. In this problem, we can assume the flow is nearly parallel far from the ends and the free surface remains flat. Please answer the following questions:

(a) Please simplify Navier Stokes equation and derive the governing equation for velocity flow field far from the ends.　(10%)

(b) List the boundary conditions and find out solution of the velocity flow field far from ends.　(10%)

(c) Find out the pressure gradient dp/dx in terms of τ_w and H .　(10%)
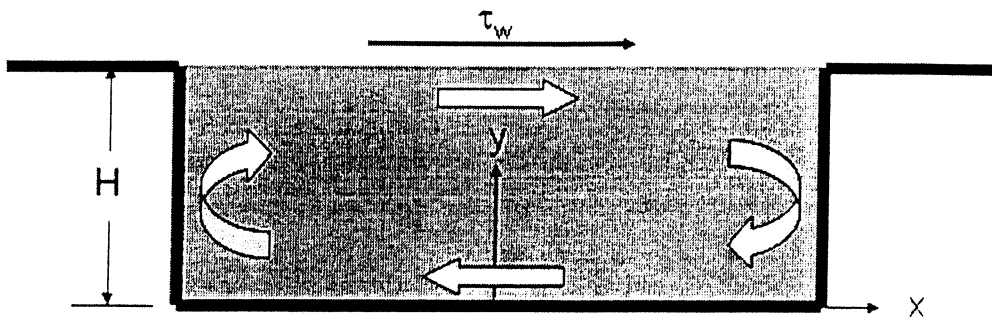

Figure 4