系所組別 奈米科技暨微系統工程研究所
考試科目 工程數學
考試日期：0307，節次：3

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1. (a) (10%) The Laplace transform of $y(t)$ is defined by

$$L(y(t)) = Y(s) = \int_0^\infty e^{-st} y(t) dt \tag{1a}$$

Show that the Laplace transform of $ty(t)$ is then given by $L(ty(t)) = -dY(s)/ds$

(b) (5%) Use the result of (a) to find the Laplace transform $L(t\dot{y})$

(c) (5%) Use the result of (a) to find the Laplace transform $L(t\ddot{y})$

2. Consider the following second-order ODE with variable coefficients

$$t\ddot{y} + (1-t)\dot{y} + y = 0, \quad y(0) = 0 \tag{2a}$$

You are asked to solve this equation by Laplace transform.

(a) (10%) By applying the results of problem 1, show that the above differential equation can be transformed to the following first-order ODE

$$(s - s^2)\frac{dY(s)}{ds} + (2-s)Y(s) = 0 \tag{2b}$$

(b) (5%) Find a solution $Y(s)$ to Eq.(2b). Hint: verify that your answer is given by

$$Y(s) = \frac{s-1}{s^2} \tag{2c}$$

(c) (5%) Find the inverse Laplace transform $L^{-1}(Y(s))$ to give the solution $y(t)$ to Eq.(2a).

3. (a) (10%) Given a 2×2 matrix

$$A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix} \tag{3a}$$

Find the eigenvalues and eigenvectors of the matrix A

(b) (10%) For the matrix A given by Eq.(3a), find a matrix X such that $X^{-1}AX$ is diagonal.

4. Consider the following line integral

$$\int_C \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r} = \int_C (F_1 dx + F_2 dy + F_3 dz) \tag{4a}$$

(a) (10%) If F is the gradient of some function f, i.e., $F = \nabla f$, show that the above line integral is path independent.

(b) (10%) Apply the above result to evaluate the following line integral

$$I = \int_C (3x^2 dx + 2yz dy + y^2 dz) \tag{4b}$$

from point $(0,1,2)$ to point $(1,-1,7)$

5. (a) (15%) Find the Fourier series expansion of the following periodic function

(背面仍有題目,請繼續作答)

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$$f(x) = \begin{cases} -k, & -\pi < x < 0 \\ k, & 0 < x < \pi \end{cases} \quad \text{and} \quad f(x+2\pi) = f(x) \tag{5a}$$

(b) (5%) Apply the above Fourier series expansion to show the following identity

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4} \tag{5b}$$