1) Given an ordinary differential equation as $x\frac{d^2y}{dx^2}+2(x-1)\frac{dy}{dx}+(x-2)y=0$

If the boundary condition is given as $y(0)=y_0$, solve for y(x).

(10%)

2) The equation for force vibration of a string is $\frac{\partial^2 y}{\partial t^2}=a^2\frac{\partial^2 y}{\partial x^2}+f(x,t)$

Find the solution for the initial conditions $y(x,0)=\frac{\partial y}{\partial t}(x,0)=0$ with boundary conditions $y(0,t)=y(L,t)=0$ when the forcing function is given by $f(x,t)=\sin(\frac{n\pi x}{L})\sin\omega t$

(20%)

3) If a vector A is dèfined as $\vec{A}=\frac{y\vec{i}-x\vec{j}}{x^2+y^2}$, solve the contour integral $\oint_C \vec{A}\bullet d\vec{r}$ along a curve C when C is a counterclockwise circle $x^2+y^2=1$

(15%)

4) Show that $\int_0^\infty \frac{\cos x}{x^2+a^2}dx=\frac{\pi}{2a}e^{-a}$ where a is a real constant..

(15%)

5) Find the general solution for the following simultaneous matrix differential equations:

(a) $t\begin{bmatrix}\frac{dy_1}{dt}\\ \frac{dy_2}{dt}\end{bmatrix}=\begin{bmatrix}2 & -1\\ 3 & -2\end{bmatrix}\begin{bmatrix}y_1\\ y_2\end{bmatrix}$ (15%)

(b) $\begin{bmatrix}\frac{dy_1}{dt}\\ \frac{dy_2}{dt}\end{bmatrix}=\begin{bmatrix}-4 & 2\\ 2 & -1\end{bmatrix}\begin{bmatrix}y_1\\ y_2\end{bmatrix}+\begin{bmatrix}t^{-1}\\ 2t^{-1}+4\end{bmatrix}$ (15%)

6) Find the Fourier series of a periodic function f(x) described by

$$f(x)=\begin{cases}-x & -L\le x<0\\ x & 0\le x<L\end{cases}$$

$$f(x+2L)=f(x)$$

(10%)