可使用計算機

Problem 1: Explain or answering the following questions. (25 Points. 5 Points each)

(a) What is the Saint-Venant's Principle? What is its major applications?

(b) Please tell us the basic assumption to formulate the equation of beam bending.

(c) Please draw a typical engineering stress-strain and a true stress-strain diagram. Labeling important information you will obtain from it. In addition, please tell us why there are difference between these two curves.

(d) What is the definition of von Mises stress? What is the von Mises failure criterion?

(e) Please define the plane strain problem. In what situation one can use plane strain to model a 3-D problem?

Problem 2: (25 Points)

As shown in Figure P2, a composite beam is made of a substrate (Young's modulus and Poisson's ratio are E_s and ν_s, respectively) and a film (Young's modulus and Poisson's ratio are E_f and ν_f, respectively). The thickness of the substrate and the film are d_s and d_f, respectively. It is reasonable to assume that $d_f << d_s$. Now, assuming that there is a residual stress σ_f existed in the film.

(a) Please draw the schematic plot on stress distribution of the beam along the thickness direction. What is the average stress existed in the substrate? (5 Points)

(b) Please draw a schematic deformation shape of the beam if σ_f is compressive and tell us why it should deform at that shape. (8 Points)

(c) Please show that the deformed radius of curvature of the beam, R, can be expressed as

$$R \approx \frac{1}{6}\frac{E_s d_s^2}{d_f \sigma_f}$$

You should state your assumption clearly! (12 Points)

Problem 3: (25 Points)

A couple $\mathbf{M_B}$ of 70 N·m is applied to a 25-mm-diameter aluminum alloy shaft as shown in Figure P3. The ends A and C of the shaft are built-in and prevented from rotating. In addition, a steel circular disk B (diameter = 100 mm, thickness = 20 mm, ρ =8000 Kg/m^3) is rigidly attached at B. Assuming that the shaft is massless. The Young's modulus and the Poisson's ratio of this aluminum alloy are 70 GPa and 0.33, respectively and this aluminum alloy can be treated as homogeneous and isotropic.

(a) Please find the angle of twist at the center **"O"** of the shaft. (13 Points)

(b) What is the shear modulus of this aluminum alloy? (4 Points)

(c) If the torque $\mathbf{M_B}$ is suddenly released. Please estimate the rotational natural frequency of this disk-shaft system. (8 Points)

Problem 4: (25 Points)

The solid circular shaft in Figure P4 is subjected to belt pulls at each end and is simply supported at the two bearings. The material has a yield strength of 250 MPa.

(a) Please draw the bending moment diagram. (5 Points)

(b) Please locate the critical location and draw its stress state. (5 Points)

(c) Determine the required diameter d of the shaft using the maximum normal stress theory together with a safety factor of 3. (10 Points)

(d) Repeat part (c) if the maximum normal stress theory is replaced by the Tresca criteria. (5 Points)

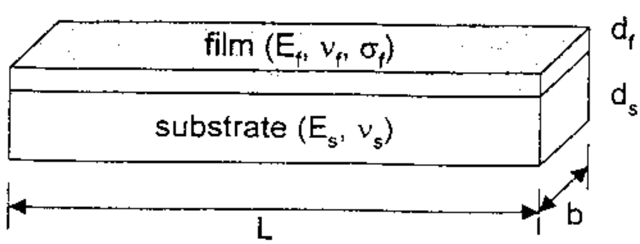
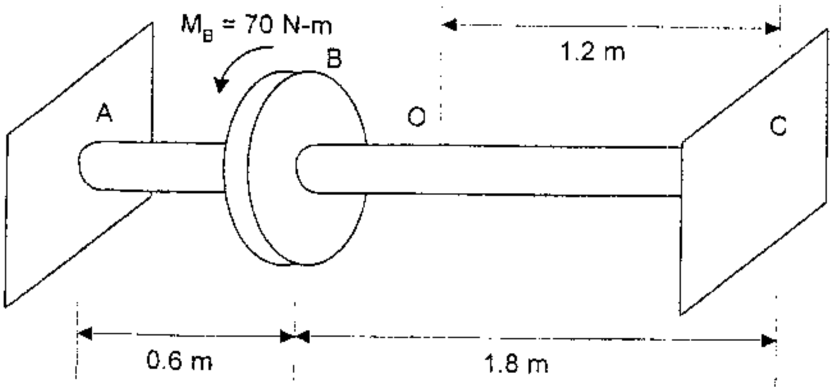


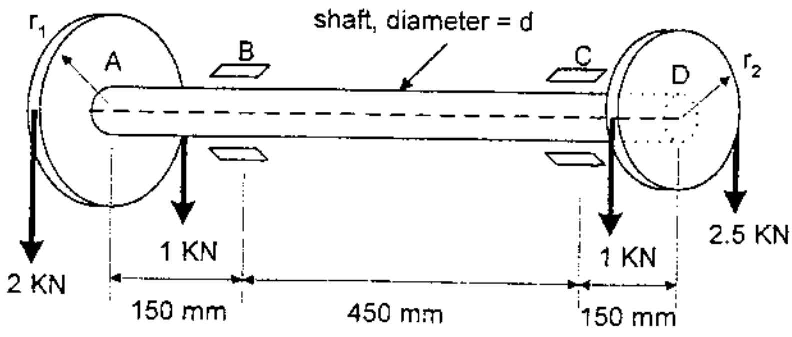
Figure P2



shaft diameter = 25 mm, massless

disk B: diameter = 100 mm, thickness = 20 mm, density = 8000 Kg/m³

Figure P3



B, C are bearings r_1 = 600 mm; r_2 = 400 mm

Figure P4