1. A particle is moving upward under the Earth's gravitational field. Assume that the particle's initial velocity is V_0 and the air friction f_r is proportional to the particle's velocity V such that $f_r = kV$ with k being a constant.
   (a) (6%) Write down the equation of motion for the particle.
   (b) (7%) Solve the above equation and find the particle's velocity $V(t)$ in terms of V_0, gravity constant g, and the constant k.
   (c) (7%) Find the maximum height of the particle (assume the initial height is zero).

2. A periodic function $f(x)$ with period 2π is expanded as a Fourier series：

$$f(x) = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos nx + B_n \sin nx$$

   (a) (7%) Show that the expansion constants can be expressed by

$$A_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx, \quad A_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx, \quad B_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx$$

   (b) (7%) Apply the above result to expand the function $f(x) = x^2$ as a Fourier series in the interval $-\pi \le x \le \pi$.
   (c) (6%) Employ the above Fourier series for $f(x) = x^2$ to verify the following summation identities

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \frac{\pi^2}{6}$$

$$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}$$

3. Consider a force field $\vec{F} = (2xy + z^3)\vec{i} + x^2\vec{j} + 3xz^2\vec{k}$, and answer the following questions：
   (a) (6%) Show that this force is conservative.
   (b) (7%) Find the potential function $\phi(x,y,z)$ for this force field.
   (c) (7%) Calculate the work done by this field upon a particle moving from position $(1,-2,1)$ to position $(3,1,4)$.

4. (20%) A particle is moving on the complex plane, whose position is represented by the complex variable $z(t) = x(t) + iy(t)$ with $i = \sqrt{-1}$ and $x,\ y \in \mathbb{R}$. If we assume that the particle's complex velocity is proportional to its position such that

$$\frac{dz}{dt} = 2iz, \quad z(0) = 1 + i,$$

   find the particle's complex position $z(t)$ as a function of time t.

5. Laplace equation in spherical coordinate system is in the form of

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial r^2} + \frac{2}{r}\frac{\partial\psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2} + \frac{\cot\theta}{r^2}\frac{\partial\psi}{\partial\theta} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2} = 0$$

（a） （10% ） We want to solve this PDE by the separation of variable $\psi(r,\theta,\phi) = R(r)S(\theta,\phi)$. Obtain the ODE for $R(r)$ and the PDE for $S(\theta,\phi)$ by assuming that the separation constant is $k = n(n+1)$, $n > 0$.（just list the ODE and PDE, and their solution is not required）

（b） （10% ） Apply the separation of variable once again by letting $S(\theta,\phi) = \Theta(\theta)\Phi(\phi)$ and using the separation constant $-m^2$. Derive the ODE's for $\Theta(\theta)$ and $\Phi(\phi)$.（just list the ODE's, while their solution is not required）