

1. Explain the meaning of curl \mathbf{A} (i.e., $\nabla \times \mathbf{A}$) and divergence \mathbf{A} (i.e., $\nabla \cdot \mathbf{A}$). (10%)
Explain $\nabla \cdot (\nabla \times \mathbf{A}) = 0$ and $\nabla \times \nabla A = 0$, where A is a scalar and \mathbf{A} is a vector (10%).
Do not derive the equation, explain them with the meaning of ∇A , $\nabla \times \mathbf{A}$, $\nabla \cdot \mathbf{A}$ themselves.
2. $\mathbf{A} = 5e^{-2z}(\rho \mathbf{a}_\rho + \mathbf{a}_z)$, determine the flux of \mathbf{A} out of the surface defined by $\rho = 1$, $0 \leq z \leq 1$, $0 \leq \phi \leq 2\pi$, where \mathbf{a}_ρ and \mathbf{a}_z are unit vectors. (20%)
3. $\mathbf{J} = r^{-3}(2\cos\theta \mathbf{a}_r + \sin\theta \mathbf{a}_\theta)$ A/m², where \mathbf{J} current density; A , ampere; m , meter.
Calculate the current passing through (1) a hemispherical shell of radius 10 cm (10%) and (2) a spherical shell of radius 5 cm (10%).
4. A conducting bar can slide freely over two conducting rails shown in the following figure. Calculate the induced voltage in the bar: (1) if the bar is stationed at $x = 5\text{cm}$ and $\mathbf{B} = 2\cos 10^5 t \mathbf{a}_z$ mWb/m² (8%); (2) if the bar slides at a velocity $\mathbf{u} = 10 \mathbf{a}_x$ m/s and $\mathbf{B} = 2 \mathbf{a}_z$ mWb/m² (7%); (3) if the bar slides at a velocity $\mathbf{u} = 10 \mathbf{a}_x$ m/s and $\mathbf{B} = 2\cos(10^5 t - x) \mathbf{a}_z$ mWb/m² (5%). Note that Wb is Weber.
5. A certain transmission line operates at $\omega = 10^7$ rad/s with $\alpha = 2$ Np/m, $\beta = 1$ rad/m, and $Z_0 = 50 + j20 \Omega$ and 2 m long. If the line is connected to a source of $V_s = 4 \angle 0^\circ$ V with $Z_s = 50 \Omega$ and terminated by a load of $10 + j20 \Omega$, determine (1) the input impedance (8%), (2) the sending-end current (7%), (3) the current at the middle of the line (5%).
 $\tanh(x \pm jy) = [(\sinh 2x) / (\cosh 2x + \cos 2y)] \pm j[(\sin 2y) / (\cosh 2x + \cos 2y)]$

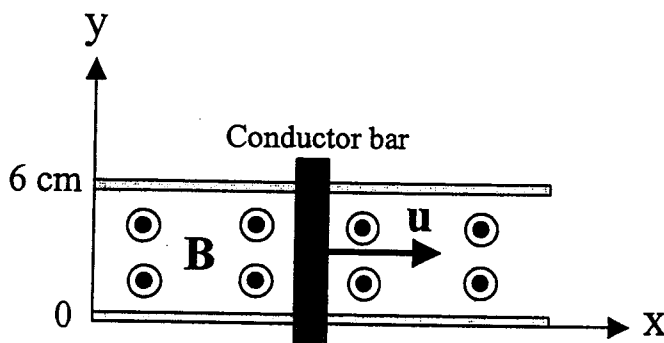


Figure of Problem 4