1. The velocity potential of a steady flow filed is given by the equation:

$$\phi = x^2 + y^2 - 2z^2$$

(a) Find the velocity components (u, v, w) for the flow filed. (5%)

(b) Show that this field represents a possible incompressible, irrotational flow. (5%)

(c) Calculate the circulation Γ about the square enclosed by x=±2 and y=±1. (5%)

(d) The temperature of the filed is described by the following expression:

$$T = x + 3xy + z^2 + 5xyz$$

Determine the time rate of change of the temperature of a fluid element as it passes through the point (1, -2, 3). (5%)

(e) Calculate the pressure difference between points (3,6,5) and (5, -3, 8) (10%)

2. Assume a spherical bubble with radius R growing in a pool of liquid. The growth rate of the bubble is $\dot{R}$.

(a) Find the velocity potential of flow filed outside the bubble as a function of coordinate r in Figure 1. (10%)

(b) Determine the pressure just outside the bubble P_{bo} when the pressure far away from the bubble is P_{∞}. (10%)

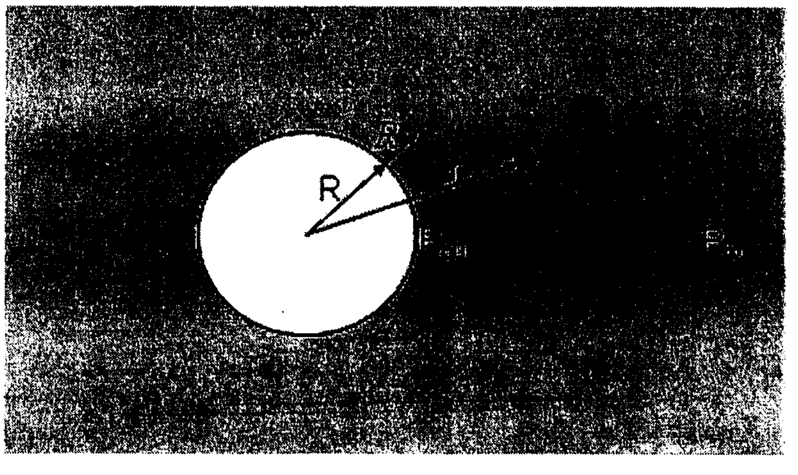

Figure 1

(背面仍有題目,請繼續作答)

3. A steady, two dimensional, incompressible pressure driven flow occurs in a slot with a moving wall, as shown in Figure 2.

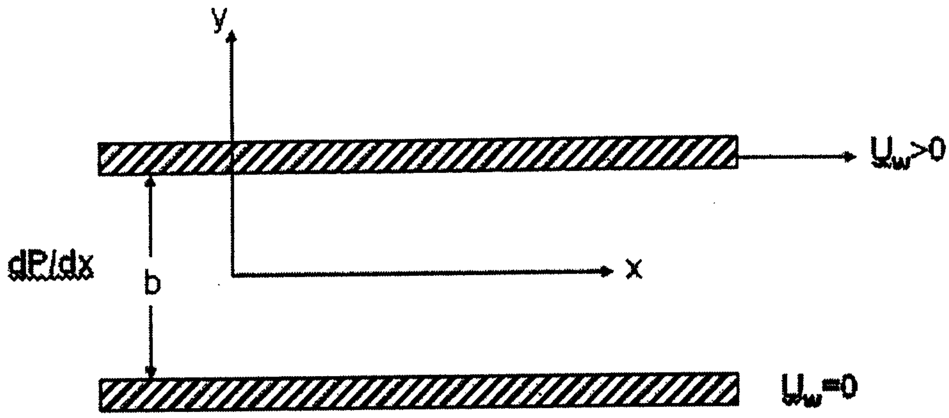

Figure 2

(a) Find the velocity distribution if the flow is laminar flow and under pressure gradient dP/dx<0. (5%)

(b) Determine the stream function for the flow. (5%)

(c) Is the flow irrotational? (5%)

(d) Determine the shear stress distribution? (5%)

(e) Discuss the maximum velocity y locations. (10%)

4 Determine the True and False in the following statements:

(a) There is no pressure gradient across the boundary layer, i.e. $\frac{\partial p}{\partial y}=0$ (5%)

(b) Outside the boundary layer, the flow can simply described as $U\frac{\partial U}{\partial x}=-\frac{1}{\rho}\frac{dp}{dx}$ (5%)

(c) For boundary layer on a surface with pressure gradient, flow separation occurs under the adverse pressure gradient, i.e. $\frac{\partial P}{\partial x}>0$. (5%)

(d) When flow separation occurs, the velocity gradient at the wall is $\left[\frac{\partial u}{\partial y}\right]_{y=0}=0$ (5%)