※ 考生請注意：本試題 □可 ☑不可 使用計算機

1. Find the general solutions of the following differential equations. (20%)

   (a) $y'=(y-1)\cot x$; (b) $y' + y = \frac{x}{y}$

2. Solve the following initial value problems: (15%)

   $5y'' + 16y' + 12.8y = 0; y(0) = 0,\ y'(0) = -2.3$

3. Let $\mathbf{x}(t)$ be a 2×1 vector. Solve the initial value problem $\frac{d}{dt}\mathbf{x}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{b}$;

   $\mathbf{x}(0) = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$, where the matrix $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ -4 & -5 \end{bmatrix}$ and the vector $\mathbf{b} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. (15%)

4. Solve the following partial differential equation for $u(x,t)$, defined on the interval $-\infty < x < \infty,\ t > 0$:

   $$\frac{\partial u(x,t)}{\partial t} = \frac{\partial^2 u(x,t)}{\partial x^2}; \quad u(x,0) = e^{-x^2}. \qquad (15\%)$$

5. Evaluate the following integral

   $$\int_0^1 e^{it}\cos at\,dt \qquad (15\%)$$

6. Let $a,b > 0$. By considering a path along the ellipse $\{a\cos t + ib\sin t \mid 0 \le t \le 2\pi\}$, evaluate

   $$\int_0^{2\pi} \frac{dt}{a^2\cos^2 t + b^2\sin^2 t} \qquad (20\%)$$