1. Let A be a symmetric tridiagonal matrix (i.e., A is symmetric and $a_{ij} = 0$ whenever $|i-j| > 1$). Let $\boldsymbol{B}$ be the matrix formed from A by deleting the first two rows and columns. Show that $\det(A) = a_{11}A_{11} - a_{12}^2 \det(\boldsymbol{B})$, where A_{11} is the cofactor of a_{11}. (10%)

2. Let A be an $m \times n$ matrix. If $\boldsymbol{B}$ is a nonsingular $m \times m$ matrix, show that $\boldsymbol{B}A$ and A have the same nullspace and hence the same rank. (10%)

3. Use the matrix exponential to solve the initial value problem $\dot{\boldsymbol{Y}} = A\boldsymbol{Y}$, $\boldsymbol{Y}(0) = \boldsymbol{Y}_0$, where $A = \begin{bmatrix} 3 & 4 \\ 3 & 2 \end{bmatrix}$ and $\boldsymbol{Y}_0 = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$. (10%)

4. Solve for the given differential equations.
   (a) $(1-x^2)y'' - 2xy' + 2y = 0, \quad -1 < x < 1.$ (7%)
   (b) $y'' - 3y' - 4y = \dfrac{e^{4x}(5x-2)}{x^3}.$ (7%)
   (c) $yy'' + (y+1)(y')^2 = 0.$ (6%)

5. Use the Laplace transform to solve Bessel's equation of order zero. (10%)
   $ty'' + y' + ty = 0;\ y(0) = 1.$

6. Solve for the integral equation. (5%)
   $f(t) = \cos(t) + e^{-2t}\int_0^t f(a)e^{2a}\,da$

7. (a) Find a Fourier series of period 6 which in the interval (1, 7) represents a function $f(x)$ taking on the constant value +1 when $1 < x < 4$ and constant value −1 when $4 < x < 7$.
   (b) Reducing the above Fourier series to the following form:
   $$f(x) = A \sum_{n\ \text{odd}} B \sin\frac{n\pi(x-1)}{3}$$
   What are the values of A and B? (12%)

8. Suppose that the analytic function $f(z)$ has a pole of order m at the point $z = a$, derive the formula for evaluating the residue $\mathbf{Res}[f(z); a]$. (8%)

9. Evaluate the following integral (15%)
   $$\int_{-1}^{1} \frac{z+1}{z^2}\,dz$$
   (a) If the path is the upper half of the circle $r = 1$,
   (b) If the path is the lower half of the circle $r = 1$,
   (c) Explain the solutions you have obtained in (a) and (b).