1. Find the general solutions of the given differential equations.
   (a) $y''+5y'=xe^{-x}\sin(3x)$. (10%)
   (b) $(2x^2+3x+1)y''+2xy'-2y=0$; $y_1(x)=x$ is a solution for x in any interval not containing -1 or $-1/2$. (10%)

2. Use the Laplace transform to solve the system. (15%)
   $x'+2x-y'=0,\ x'+y+x=t^2;\ x(0)=y(0)=0$.

3. Please show that the eigenvalues of a Hermitian matrix are real. (10%)

4. Let $\mathbf{D}$ be the differentiation operator on $\mathbf{P}_3$ (where $\mathbf{P}_n$ denotes the set of all polynomials of degree less than n) and let $\mathbf{A}=\{\mathrm{p}\in\mathbf{P}_3\mid \mathrm{p}(0)=0\}$. Please show that $\mathbf{D}:\mathbf{A}\to\mathbf{P}_3$ is one-to-one but not onto. (10%)

5. Let $\mathbf{B}=\begin{bmatrix}8&0&0&0\\1&8&0&0\\0&0&1&1\\0&0&0&1\end{bmatrix}$, please find $\mathbf{B}^{\frac{1}{3}}$. (10%)

6. (a) Find the Fourier integral representation of the function
   $$f(t)=\begin{cases}0 & -\infty<t\le -1\\ 1+t & -1\le t\le 0\\ 1-t & 0\le t\le 1\\ 0 & 1\le t<\infty\end{cases}$$
   and express the integral which approximates this function for frequencies between 0 and ω_0 in terms of the sine-integral function. (15%)
   (b) Use the result of (a), show that
   $$\frac{\pi}{2}=\int_0^{\infty}\frac{1-\cos\omega}{\omega^2}d\omega.\quad (5\%)$$

7. Evaluate $\oint_c(\bar{z}-a)^{-1}(b-\bar{z})^{-1}(z^2+z^{-2})dz,\ 0<|a|<r<|b|$, where c is the positive orientation of the circle $\{z;|z|=r\}$. (15%)