1. Please find the least squares solution to the system described by, $-2y_1+3y_2=1$, $2y_1-y_2=2$, and $y_1+y_2=3$ (15%)

2. Let λ an eigenvalue of an $n\times n$ matrix A, and let X be an eigenvector belonging to λ. Please show that e^{λ} is an eigenvalue of e^{A} and X is an eigenvector of e^{A} belonging to e^{λ}. (15%)

3. Find the general solutions of the given differential equations.

   (a) $(y^2+1)dx = y\sec^2 x dy$. (10%)

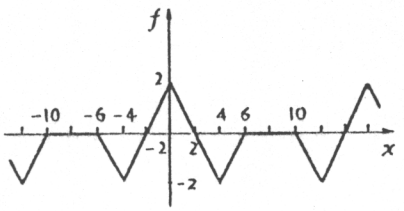
   (b) $y''-y'-12y = 2\sinh^2 x$. (10%)

4. Given that $t(1-t)y''+2y'+2y = 6t$; $y(0)=0$, $y(2)=0$

   (a) Identify the type of the problem for the given equation and conditions. (5%)

   (b) Please use the Laplace transform to solve the problem. (15%)

5. Find the Fourier series of the periodic function f, the graph of which is shown in the following figure. (15%)



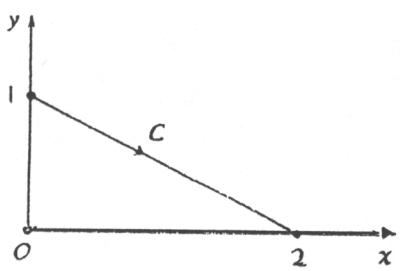
6. Knowing that the **ML bound** describes

$$\left|\int_C f(z)dz\right| \le ML$$

where $|f(z)| \le M$ on C and L is the length of C. Find the **ML bound** of the following integral

$$I = \int_C \frac{e^z}{z^2}dz$$

where C is the straight line as shown. (15%)



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