

本試題是否可以使用計算機： 可使用， 不可使用（請命題老師勾選）

- (10%) 1. Solve the following initial value problems:

$$y' = 2e^x y^3, y(0) = 0.5$$

- (10%) 2. Solve the following initial value problems:

$$y'' - 4y' + 4y = 0, y(0) = 0, y'(0) = -3$$

- (15%) 3. Solve the following initial value problems:

$$y''' - y'' - 4y' + 4y = 6e^{-x}$$

- (15%) 4. Let $f(n) = \mathbf{v}^T \mathbf{A}^n \mathbf{v}$, where $(\cdot)^T$ denotes transpose, n is a positive integer,

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 1 & 3 \end{bmatrix}, \text{ and } \mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}. \text{ Please evaluate } \lim_{n \rightarrow \infty} \frac{\log_2 f(n)}{n}.$$

- (20%) 5. Find solutions $y(r, \theta)$ of the following boundary value problem for

$0 \leq \theta \leq \pi$ and $1 < r < \infty$ by separation of variables:

$$r \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} y(r, \theta) \right) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} y(r, \theta) \right) = 0,$$

$$y(1, \theta) = \cos \theta, \quad 0 \leq \theta \leq \pi.$$

- (15%) 6. Expand $1/\sin z$ about $z = 0$ in $\pi < |z| < 2\pi$

- (15%) 7. Using the residue theorem to evaluate

$$\int_0^\pi \frac{\cos \theta}{\cos \theta - \cos \alpha} d\theta, \quad \alpha \text{ is a constant}$$