

本試題是否可以使用計算機: 可使用, 不可使用 (請命題老師勾選)

1. Find the general solution of the differential equation:

(a) $y'' - 2y' + y = 3x + 25 \sin(3x)$ (10pts)

(b) $y'' + y' - 6y = 50xe^{2x}$ (10pts)

2. Use the Laplace transform to solve the initial value problem:

(a) $y' + 2y = e^{-t}; y(0)=1$ (5 pts)

(b) $y'' - 4y' + 4y = \cos(t); y(0)=1, y'(0) = -1$ (5 pts)

(c) $y'' - 5y' + 6y = e^{-t}; y(0)=0, y'(0) = 2$ (5 pts)

3. Let

$$f(z) = \begin{cases} z/|z| & \text{for } z \neq 0 \\ 0 & \text{for } z = 0 \end{cases}$$

At what points do the real and imaginary parts of f satisfy the Cauchy-Riemann equations? (15%)

4. Let

$$f(z) = \frac{1}{z^2(z+2i)}$$

(a) Find a Laurent series in powers of z which converges to f and specify the region on which the series converges. (8%)(b) Find another Laurent series in powers of z which converges to f and specify the region on which the series converges. (7%)5. (a) Find the Fourier cosine transform of $f(x) = e^{-x}$. ($x > 0$) (10%)(b) Use the above result to evaluate $\int_0^{\infty} \frac{dx}{(x^2+1)^2}$. (10%)6. Find the solution $u(x,t)$ of the following boundary-value problem: (15%)

$$\frac{\partial^2 u}{\partial x^2} - 2 = \frac{\partial u}{\partial t}, \quad 0 < x < 1, t > 0$$

$$u(0,t) = 0, \quad u(1,t) = 2, \quad t > 0$$

$$u(x,0) = x^2 + x + \sin^3 \pi x, \quad 0 < x < 1.$$