

本試題是否可以使用計算機： 可使用， 不可使用（請命題老師勾選）

考試日期：0301，節次：3

1. Find the curve  $y(x)$  that passes through  $(1, 0.5)$  and is such that at each point  $(x, y)$  the intercept of the tangent on the  $y$ -axis is equal to  $2xy^2$ .  
(15%)

2. Solve the following initial value problems.

(a)  $x^2 y'' - 4xy' + 4y = 0$ ,  $y(1) = 4$ ,  $y'(1) = 13$ . (10%)

(b)  $(x^2 D^2 - 5xD + 8)y = 0$ ,  $y(1) = 5$ ,  $y'(1) = 18$ . (10%)

3. Compute

$$\int_0^{\infty} \frac{\sin^2 x}{x^2} dx \quad (15\%)$$

4. What is the order of the pole at  $z = 0$  of the following function? Why?

$$f(z) = \frac{1}{(2 \cos z - 2 + z^2)^2} \quad (15\%)$$

5. Find the Singular-Value Decomposition (SVD) of the matrix

$$A = \begin{bmatrix} 5 & 0 & 1 & 0 \\ 1 & 0 & 5 & 0 \\ 0 & 4 & 0 & -4 \end{bmatrix}. \quad (15\%)$$

6. Let  $f(t) = e^{-|t|}$ , and  $g(t) = \begin{cases} 1, & -1 \leq t < 1 \\ 0, & \text{otherwise} \end{cases}$  (20%)

(a) Compute  $y(t) = f(t) * g(t)$ , where  $*$  denotes convolution.

(b) Find the Fourier transform of  $y(t)$ .