

※ 考生請注意：本試題 可 不可 使用計算機

1. (10%) Use of residue methods to evaluate

$$\int_{-\infty}^{+\infty} \frac{\cos mx}{x^2 + x + 1} dx, \quad m = 1, 2, 3, \dots$$

2. (20%) Use of contour integral methods to evaluate

$$\int_{-\infty}^{+\infty} \frac{1}{x\sqrt{x^2-1}} dx$$

3. (15%) Solve $x \frac{dy}{dx} + y = x^2 y^2$

4. (20%) Solve the following differential equations.

(a) $2y'' - 5y' - 3y = 0$

(b) $y'' - 10y' + 25y = 0$

5. (20%) Define the Frobenius norm $\|A\|_F$ of an $m \times n$ matrix $A = [a_{ij}]$ as

$$\|A\|_F = \left(\sum_{i=1}^m \sum_{j=1}^n a_{ij}^2 \right)^{1/2}. \quad \text{Let } B = \begin{bmatrix} 5 & 8 & 10 \\ 11 & 8 & -2 \\ 13 & 4 & 2 \\ 3 & 12 & 6 \end{bmatrix}.$$

(a) Find the matrix C of rank 1 that minimizes $\|B - C\|_F$.

(b) Find the matrix D of rank 2 that minimizes $\|B - D\|_F$.

6. (15%) For the rectangular region $R = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 1\}$, solve the

Dirichlet problem $\nabla^2 u(x, y) = 0$, with the boundary conditions $u(0, y) = u(x, 0) = 0$, $u(x, 1) = \pi$, and $u(1, y) = 2\pi$.