系所組別： 資訊工程學系
考試科目： 計算機數學

※ 考生請注意：本試題 ☑可 □不可 使用計算機

Part I.

Linear Algebra (50%)

1. Let $\mathbf{A}=\begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -2 \\ 2 & -1 & -1 \end{bmatrix}$,

   (a) Find the characteristic polynomial and Eigenvalues of **A**. (8%)

   (b) Find the eigenvectors of **A**. (8%)

   (c) Is matrix **A** diagonalizable? If yes, calculate the diagonal matrix. (8%)

   (d) Let L be the linear operator mapping $\mathbb{R}^3$ into $\mathbb{R}^3$ defined by L(x)=**A**x and let $\mathbf{v}_1=\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{v}_2=\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$, $\mathbf{v}_3=\begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$. Find the transition matrix V corresponding to a change of basis from $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ to $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$, and use it to determine the matrix B representing L with respect to $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$. (10%)

2. Given the vectors $\mathbf{x}_1=\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$, $\mathbf{x}_2=\begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}$, $\mathbf{x}_3=\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

   (a) Are they linearly dependent? Do they form a basis for $\mathbb{R}^3$? Explain and prove your answers. (10%)

3. The data set is:

x	-1	0	1	2
y	0	1	3	9

   (a) Find the best least squares fit by a quadratic polynomial, $P(x) = C_1 + C_2 x + C_3 x^2$. (6%)

(背面仍有題目,請繼續作答)

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Part II.

Discrete Mathematics (50%)

一、單選題

1. (5%) The probability of each summand is a multiple of 3 in all compositions of 18 is
(A). $1/3^{11}$.
(B). $1/3^{12}$.
(C). $1/2^{11}$.
(D). $1/2^{12}$.
(E). None of the above.

2. (5%) Which statement is **NOT** correct?
(A). The coefficient of x^5 in $(1 - 2x)^{-7}$ is $(32)\binom{11}{5}$.
(B). $\sum_{k=0}^{20}(-1)^k \binom{20}{20-k}(20-k)^{15} = 0$
(C). If $|A| = |B| = 6$, there are 6! functions f: A→B are invertible.
(D). The sequence generated by $f(x) = \frac{1}{3-x}$ is $(-\frac{1}{3})$, $(-\frac{1}{3})^2$, $(-\frac{1}{3})^3$, $(-\frac{1}{3})^4, \ldots$.
(E). None of the above.

3. (10%) Suppose $S(n)$ is a predicate on natural numbers, n, and suppose $\forall k \in \mathbb{N}, S(k) \rightarrow S(k+2)$ hold. Which one following statements **NEVER** hold?
(A). $\forall n \geq 0\ S(n)$.
(B). $\forall n \geq 0\ \neg S(n)$.
(C). $[\exists n\, S(2n)] \rightarrow \forall n\, S(2n+2)$.
(D). $(\forall n \leq 100\ \neg S(n)) \wedge (\forall n > 100\ S(n))$.
(E). None of the above.

二、 計算題

1. (15%) Let $\Sigma = \{0, 1, 2, 3, 4\}$. For $n \geq 1$, let a_n count the number of string in Σ^n containing an odd number of 1's. Find and solve a recurrence relation for a_n.

2. (15%) Find the number of ways to arrange the letters in LAPTOP so that none of the letters L, A, T, O is in its original position and the letter P is not in the third or sixth position.