

系所組別： 資訊工程學系

考試科目： 計算機數學

考試日期： 0223，節次： 3

※ 考生請注意：本試題不可使用計算機

## 一. Linear Algebra (50%)

## 1. True or False (15%. 3 pts each)

For each of the statements that follows, answer **true** if the statement is always true and **false** otherwise.

(a) If  $A$  and  $B$  are  $n \times n$  matrices that have the same rank, then the rank of  $A^2$  must equal the rank of  $B^2$ .

(b) Let  $L: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be a linear operator, and let  $A$  be the standard matrix representation of  $L$ . If  $L^2$  is defined by

$$L^2(\mathbf{x}) = L(L(\mathbf{x})) \text{ for all } \mathbf{x} \in \mathbb{R}^2$$

then  $L^2$  is a linear operator and its standard matrix representation is  $A^2$ .

(c) If  $L_1$  and  $L_2$  are both linear operators on a vector space  $V$ , then  $L_1 + L_2$  is also a linear operator on  $V$ , where  $L_1 + L_2$  is the mapping defined by

$$(L_1 + L_2)(\mathbf{v}) = L_1(\mathbf{v}) + L_2(\mathbf{v}) \text{ for all } \mathbf{v} \in V$$

(d) If  $N(A) = \{\mathbf{0}\}$ , then the system  $A\mathbf{x} = \mathbf{b}$  will have a unique least squares solution.

(e) If  $\{\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_k\}$  is an orthonormal set of vectors in  $\mathbb{R}^n$  and

$$U = \{\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_k\}$$

then  $UU^T = I_n$  (the  $n \times n$  identity matrix).

## 2. (20%. 10 pts each)

Let

$$\mathbf{x} = \begin{bmatrix} 4 \\ 4 \\ -4 \\ 4 \end{bmatrix} \text{ and } \mathbf{y} = \begin{bmatrix} 4 \\ 2 \\ 2 \\ 1 \end{bmatrix}$$

(a) Determine the angle between  $\mathbf{x}$  and  $\mathbf{y}$ .

(b) Determine the distance between  $\mathbf{x}$  and  $\mathbf{y}$ .

## 3. (15%)

Let

$$\mathbf{u}_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \mathbf{u}_2 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

and let  $L$  be the linear operator that rotates vectors in  $\mathbb{R}^2$  by  $45^\circ$  in the counterclockwise direction. Find the matrix representation of  $L$  with respect to the ordered basis  $\{\mathbf{u}_1, \mathbf{u}_2\}$ .

(背面仍有題目,請繼續作答)

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## 二、Discrete Mathematics (50%)

4. (20%) Solve  $a_n = 3a_{n-1}^2, a_0 = 1$ .
5. (20%) Consider the permutations of 1, 2, 3, 4. The permutation 1432 is said to have one *ascent* (since  $1 < 4$ ) and two *descents* (since  $4 > 3$  and  $3 > 2$ ). Suppose a permutation of  $1, 2, 3, \dots, m$  has  $k$  ascents, for  $0 \leq k \leq m-1$ . How many descents does the permutation have?
6. (10%) Given 8 Perl books, 17 Python books, 6 Java books, 12 SQL books, and 20 Objective -C books, how many of these books must we select to insure that we have 10 books dealing with the same computer language?