國立成功大學 76 學年度 資訊工程研究所 考試(工程數學 試題) 共 1 頁 第 1 頁

NOTICE To any part of the following problems, if your solution or proof is incomplete, that part will get zero point.

1. Given is the recurrence relation $F_n = F_{n-1} + F_{n-2}$ for n = 2, 3,....., with $F_0 = F_1 = 1$.

   (1) Find the closed-form solution. (6%)

   (2) Show that $\sum_{n=0}^{N} F_n^2 = F_{N+1}F_N$. (6%)

   (3) Show that $\sum_{n=1}^{2N} (-1)^{n+1} F_n = -F_{2N-1}$. (6%)

2. Let A_k, L_k, and U_k be square matrice and let $A_k = L_k U_k$ for k = 1, 2,...., where L_k's are lower triangular and U_k's are upper triangular. Both L_k and U_k are nonsingular. Also let $A_n = U_{n-1}L_{n-1}$ for n = 2, 3,... .

   (1) Show that A_n and A_1 have the same eigenvalues. (6%)

   (2) Let $Q_n = L_1 L_2 \ldots L_n$ and $R_n = U_n U_{n-1} \ldots U_1$. Show that $Q_n R_n = A_1^n$. (6%)

   (3) Assume that $\lim_{n\to\infty} R_n = R$ exists and is nonsingular. Show that $\lim_{n\to\infty} A_n$ exists and is lower triangular. (6%)

3. Let X be a value representing the outcome of a random experiment, where the density function is given as

$$f_X(x) = \begin{cases} \exp(-x), & \text{for } x \geq 0 \\ 0, & \text{elsewhere} \end{cases}$$

   (1) Find the probability that, in five performances of the experiment, at least three values of X will be greater than 1/2. (7%)

   (2) If $Y = (X + 1)^2$, find the probability that Y lies between 5 and 10. (7%)

4. The density function of random variable X is given as

$$f_X(x) = \begin{cases} (x+1)/8, & \text{for } -1 \leq x \leq 3 \\ 0, & \text{elsewhere} \end{cases}$$

   (1) Let Y be a random variable which represents only those values of X that fall outside the interval (1, 2). Find the density function, mean, and variance of Y. (10%)

   (2) Let W be another random variable defined by $W = |Y|$. Find the density function, mean, and variance of W. (10%)

5. Let the Gamma function be denoted by $\Gamma(x)$.

   (1) Show that $\Gamma(1/2) = \pi^{1/2}$. (10%)

   (2) Express $\Gamma(9/2)$ in terms of $\Gamma(1/2)$. (5%)

6. Show that $\int_{-\infty}^{\infty} [(\sin t)/t]\, dt = \pi$. (15%)