資訊工程研究所

PART I.

1. Let the two random variables X and Y have the the joint p.d.f. (probability density function) described as follows

(x,y)	(0,0)	(0,1)	(1,0)	(1,1)	(2,0)	(2,1)
$f(x,y)$	$\frac{2}{18}$	$\frac{3}{18}$	$\frac{4}{18}$	$\frac{3}{18}$	$\frac{4}{18}$	$\frac{2}{18}$

and $f(x,y)$ is equal to zero elsewhere. (a) Find the two marginal p.d.f. (4%) (b) Find the two conditional means. (4%) (c) Explain whether X and Y are independent or not. (2%)

2. Suppose X and Y are two random variables so that $E(X)=E(Y)=0$, $\sigma^2(X)=\sigma^2(Y)=1$. show that
   (a) $E^2(XY)\le 1$ (4%)
   (b) $E(XY)=1$ if and only if $P(Y=X)=1$. (3%)
   (c) $E(XY)=-1$ if and only if $P(Y=-X)=1$. (3%)
   where E and σ is the expected value and the variance, respectively.

3. Assume that X and Y are independent random variables with the following p.d.f.

$$X:\quad f(x)=\begin{cases}\frac{x}{2} & 0\le x\le 2\\ 0 & \text{elsewhere}\end{cases}$$

$$Y:\quad g(y)=\begin{cases}\frac{y^3}{64} & 0\le y\le 4\\ 0 & \text{elsewhere}\end{cases}$$

Calculate the p.d.f. of $W=XY$. (10%)

4. Suppose $f(z)=\dfrac{2z^2+5}{(z+1)^2(z^2+1)}$, where $z=x+iy$ is a complex variable. Consider five contours as shown in Fig. 1. Find the values of the following integrals:
   (a) $\int_{C_1} f(z)dz$ (b) $\int_{C_2} f(z)dz$ (c) $\int_{C_3} f(z)dz$ (d) $\int_{C_4} f(z)dz$
   (e) $\int_{C_4\cup C_5} f(z)dz$
   where $C_4\cup C_5$ represents the union of C_4 and C_5. (10%)

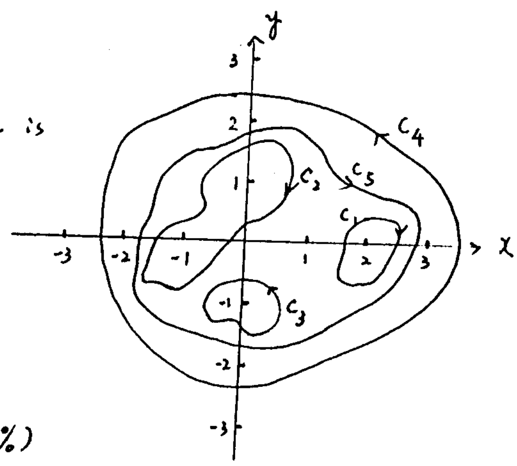

Fig. 1

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PART II

1. Let A={1,2,3} define a relation R on A such that none of reflexive, irreflexive, symmetric, anti-symmetric or transitive be hold. (5%)
2. Define nRm if and only if nm>=0 for any integers n and m. Is R an equivalence relation? Why? (5%)
3. Let $\Sigma=\{0,1\}$ design a (deterministic) finite state machine that accepts the set of all strings over Σ which ends with 100. (5%)
4. Let Q^*_+ be the set of positive rational numbers, and $a\oplus b=ab/2$ for $a,b=Q^*_+$. Prove that $<Q^*_+,\oplus>$ is a group. (5%)
5. Find the solution for the recurrence system:

$$\begin{cases} a_0=0,\ a_1=1 \\ a_n=a_{n-1}+a_{n-2} \text{ for } n>1 \end{cases}$$

and show that the solution is correct by mathematical induction. (10%)

Part III.

1. Consider the system of homogeneous equations

$$x_1-2x_2+x_3-x_4+x_5=0$$
$$3x_1-6x_2+x_3+2x_4+2x_5=0$$
$$-x_1+2x_2+2x_3-2x_4-13x_5=0$$
$$-2x_1+4x_2-3x_3+3x_4+2x_5=0$$

(a) Find the dimension of the null space of this system. (5%)
(b) Find a basis for the null space of this system. (5%)

2. Let $R^3 -> R^3$ be given by
$T(x,y,z)=(2x+z,\ 3y,\ x+2z)$.
(a) Find the eigenvalues of T. (5%)
(b) Find a basis β for R^3 such that the matrix $[T]_\beta$ of T relative to β is diagonal. (5%)

3. Let A be an nxn matrix with entries in C. Prove that if $||A\vec{x}||_2=||\vec{x}||_2$, for every $\vec{x}\in C^n$, then $(A\vec{x},A\vec{y})=(\vec{x},\vec{y})$, for every $\vec{x}$ and $\vec{y}$ in C^n, where $||\vec{x}||_2=(\vec{x},\vec{x})^{1/2}$ and $(\vec{x},\vec{y})=\vec{x}^H\vec{y}$. (10%)

Hint: Consider $(\vec{x}+\vec{y},\vec{x}+\vec{y})-(\vec{x}-\vec{y},\vec{x}-\vec{y})$ and
$(\vec{x}+i\vec{y},\vec{x}+i\vec{y})-(\vec{x}-i\vec{y},\vec{x}-i\vec{y})$.