

一. 微積分: (共 50%)

1. (a) Please transform the following equation into one in polar coordinate. (5%)

$$x^2 + y^2 = 2x$$

- (b) Plot the graph in polar coordinate. (5%)

2. The size of the world population in 1977 was approximately 3.8 billion. If it continues to grow at the rate of 2% a year, what will be the size of population in the year of 1990. (10%)

3. In the following expression, please determine the given limits

(a) $\lim_{x \rightarrow \infty} \frac{x^3}{e^x}$

(b) $\lim_{x \rightarrow -\infty} x^5 e^{-x}$

(c) $\lim_{x \rightarrow \infty} \frac{(1.01)^x}{x^{50}}$

(d) $\lim_{x \rightarrow \infty} \frac{\ln x}{x^3}$

(totally 10%)

(e) $\lim_{x \rightarrow \infty} \frac{\log_2 x}{\log_3 x}$

4. Find the derivative of the given functions:

(a) $4x^3 - \frac{2}{x^2}$

(b) $\frac{\sin x}{x^2 - 4x}$

(c) $\sqrt{1+x^2}$

Find the normal line and the tangent line to the following graph.

(d) $y = 3x^2 - 2x$ at $(2, 8)$

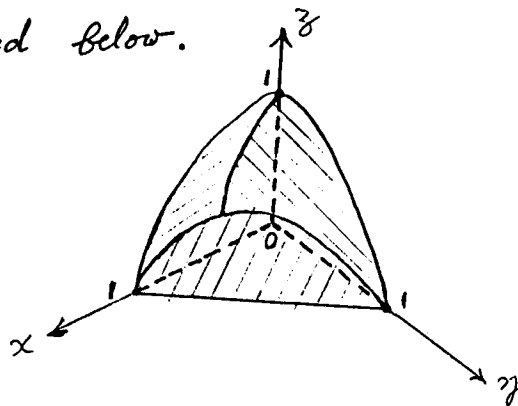
(e) $y = \frac{2x+3}{x-1}$ at $(0, -3)$

(totally 10%)

5. Please find the volume of the region in space bounded by the surface $z = 1 - x^2 - y^2$, on the sides by the planes $x=0$, $y=0$, $x+y=1$, and below by the plane $z=0$.

The region is sketched below.

(10%)



二. 線性代數:(共 50%)

1. (a) Verify whether the vector set given below is a basis for \mathbb{R}^3 ?

$$\mathbf{x}_1 = (1, 1, 1)$$

$$\mathbf{x}_2 = (1, 2, 3)$$

$$\mathbf{x}_3 = (0, 1, 0)$$

(b) Express the vector $\mathbf{x} = (2, 1, 3)$ as a linear combination of the above vector set.

(totally 10%)

2. If

$$|A| = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = -4$$

find the determinants of the following matrices:

(a) $B = \begin{bmatrix} a_3 & a_2 & a_1 \\ b_3 & b_2 & b_1 \\ c_3 & c_2 & c_1 \end{bmatrix}$ (3%)

(b) $C = \begin{bmatrix} a_1 & a_2 & 2a_3 \\ b_1 & b_2 & 2b_3 \\ c_1 & c_2 & 2c_3 \end{bmatrix}$ (3%)

(c) $D = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1+4c_1 & b_2+4c_2 & b_3+4c_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$ (4%)

3.

Let $X = 2i + j + 2k$ and $Y = 3i - j - 3k$

(a) Please get $X \times Y$ (the cross product) (5%)

(b) If $Z = i + 2j + 3k$,

please get $X \cdot (Y \times Z)$. (5%)

4.

(a) How do you define a nonsingular matrix to be orthogonal? (4%)

(b) Given $Q(X) = 2x^2 + 2xy + 2y^2$, $X = (x, y)^T$,

let $Q(X) = X^T A X$, please find the symmetric matrix A .

(4%)

③ Let $\underline{B} = \underline{P}^T \underline{A} \underline{P}$ for an orthogonal matrix \underline{P} , such that $Q'(\underline{Y}) = \underline{Y}^T \underline{B} \underline{Y}$, $\underline{Y} = \underline{P} \underline{X}$, is equivalent to Q . Please find \underline{P} and \underline{B} . (8%)

④ Let $Q(\underline{X}) = 9$, what is the resulted equation of $Q'(\underline{Y})$? (4%)