1. (20%) Suppose that it was shown that the lower bound of Problem A is $O(n^a)$ and up to now, the best algorithm, $\mathfrak{A}$, to solving A is of the worst case time complexity $O(n^b)$.
   (a) Under what condition can we say that $\mathfrak{A}$ is an optimal algorithm ?
   (b) Suppose that $a = 1$ and $b = 2$. What can you say about $\mathfrak{A}$ and the lower bound found for problem A ?

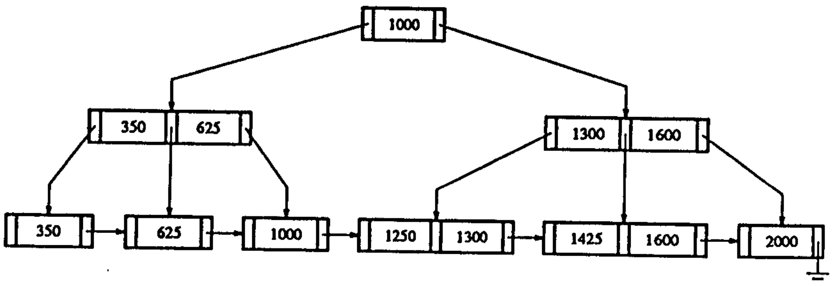
2. (20%) Find the solution of the following recurrences :
   (1) $T(n) = 2\,T(n) + O(n)$
   (2) $T(n) = T((1-a)n) + O(n)$ $\quad 0 < a < 1$
   In fact, (1) and (2) are the time complexity formulas for the divide – and – conquer and prune – and – search strategies, respectively. Use quicksort and binary search, respectively, to explain these two strategies.

3. (20%) Assume that you need to use an algorithm to sort a number of records in a machine.
   (a) If there are two restrictions in your work:
   (1) only comparison–based sorting algorithms can be used,
   (2) the sorting is performed in a serial machine.
   What is the theoretically best computing time in terms of order of complexity you may achieve? (2%) Prove it. (10%)
   (b) Give two suggestions to break the lower bound in (a) if the two restrictions are released. Briefly explain your suggestions. (8%)

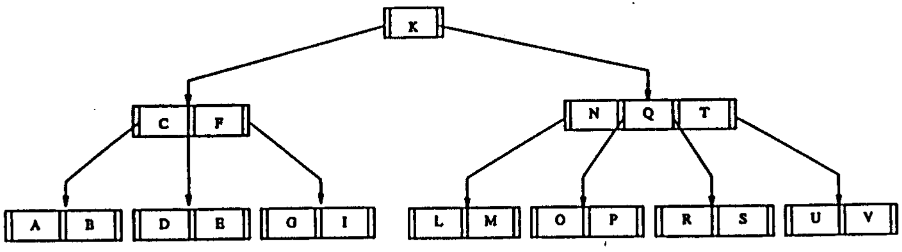
4. (10%) A *bipartite graph* $G = (V, E)$ is an undirected graph whose vertices can be partitioned into two disjoint sets V_1 and $V_2 = V - V_1$ with the properties (i) no two vertices in V_1 are adjacent in G and (ii) no two vertices in V_2 are adjacent in G. Write an algorithm to determine whether a graph G is bipartite. What is the time complexity of your algorithm.

5. (18%) Suppose that there are only two types of access requests that a user may issue to a file:
   (1) Are there any records in the file that satisfy the <selection condition>?
   (2) How many records are there in the file that satisfy the <selection condition>?
   Give a proper choice among the multilist file, inverted file, and B–tree organizations for the following situations. Explain your choice.
   (a) Most (say 90%) of the requests belong to type (1).
   (b) Most of the requests belong to type (2).
   (c) Assume that the index you may use is not limited to those given above and that you may combine different indexing mechanisms in one file. What indexing mechanism will you suggest to use if the numbers of two types of requests are about the same? Why?

6. (12%) Answer the following B–tree questions.
   (a) Redraw the following B^+–Tree of order 3 after the insertion of "1500".



(b) Redraw the following B–Tree of order 5 after the deletion of "B".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