微積分

1. (a) Please sketch the cardioid
$$r = (1 + \cos\theta), \qquad (6\%)$$
(b) Please find the surface area generated when the cardioid revolved about the X-axis. (7%)

2. Please determine the convergence or divergence of the following series: (3% each, 15% total)

(a) $\sum_{n=1}^{\infty} \frac{3n^2+3n}{n^4+2}$

(b) $\sum_{n=1}^{\infty} \frac{2^{3n+1}}{n!}$

(c) $\sum_{n=1}^{\infty} \frac{n!+2^n}{(n+2)!}$

(d) $\sum_{n=1}^{\infty} \frac{n!}{3^{(n^2)}}$

(e) $\sum_{n=1}^{\infty} \frac{1}{\ln(n^2+4n)}$

3. (a) Please find the equation of the parabola with vertex at (-5,2) and focus at (-5, 0) (6%)
(b) Please compute the area between the parabola and the X-axis. (6%)

4. If the length of an edge of a cube increases at a rate of 1 in/sec, Please find the (instantaneous) rate of increase of the volume when
(a) the edge is 2 in long.
(b) the edge is 5 in long.
(5% each, please give the derivation, 10% total)

線性代數:

1. (1) Please find a real symmetric matrix C such that

$Q = X^T C X$, where Q equals

(a) $6x_1^2 - 4x_1x_2 + 2x_2^2$

(b) $14x_1x_2 - x_2^2$

(c) $(x_1 + x_2 + x_3)^2$ (4% each, 12% total)

(2) Please verify whether the above Q and its corresponding C are positive definite in (a), (b) and (c).
(2% each, 6% Total)

2.
$$A = \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$$

(a) Please find the eigenvalues of A. (5%)

(b) Please find the eigenvectors of A. (5%)

(c) What are the eigenvalues of A^2. (5%)

3. (a) Please find the inverse matrix of
$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 5 & 5 & 1 \end{pmatrix}$$
if possible. (5%)

(b) If $B = AX$,

please get the solution of X, given $B = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$ (5%)

(c) What is the minimal polynomial for the matrix A? (7%)