Part I : Linear Algebra

1. For the following system of three equations in four unknowns:

$$
\begin{aligned}
7x_3 + 14x_4 &= -7 \\
2x_1 - 8x_2 + 4x_3 + 18x_4 &= 0 \\
3x_1 - 12x_2 - x_3 + 13x_4 &= 7
\end{aligned}
$$

(a) Find a row-echelon form. (5%)

(b) Determine the solution(s) of the original system of equations. (5%)

2. For the following symmetric matrix A, (10%)

$$A = \begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}$$

(a) Find a matrix P such that $P^{-1}AP$ is diagonal.

(b) Determine an orthogonal matrix $\hat{P}$, such that $(\hat{P})^{-1}A\hat{P}$ is diagonal.

3. Define the following terms:

(a) Markov process (5%)

(b) Linear transformation (5%)

4. Given a straight line in 2-space that passes through the two points $P(-2,-3)$ and $Q(5,1)$

(a) Find a vector equation for the straight line. (5%)

(b) Determine a direction vector for the straight line. (5%)

5. Let $L: R^3 \to R^2$ be defined for $x = [x_1, x_2, x_3]^t$ by

$$L(x) = \begin{bmatrix} 3x_1 - x_2 + 2x_3 \\ 2x_1 + 4x_2 - x_3 \end{bmatrix}$$

Let $S = \{v_1, v_2, v_3\}$ and $T = \{w_1, w_2\}$, where $v_1 = [1,1,0]^t$, $v_2 = [1,0,1]^t$, $v_3 = [1,1,1]^t$, $w_1 = [1,1]^t$, and $w_2 = [-1,0]^t$

(a) Determine the matrix A of L with respect to bases S and T. (5%)

(b) Compute $L(x)$ for $x = [4,2,1]^t$. (5%)

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Part II : Discrete Mathematics

1. (a) Find the inverse of the function $f: R \to R^+$ defined by $f(x) = e^{2x+5}$. (4%)

   (b) Show that $f \circ f^{-1} = 1_{R^+}$ and $f^{-1} \circ f = 1_R$. (4%)

   (c) Graph f and f^{-1} on the same set of axes. (2%)

2. Consider the following experiments:

   Experiment 1 : Flip a coin.

   Experiment 2 : Roll two fair dice.

   (a) Construct a single probability space to model the compound experiment consisting of Experiment 1 followed by Experiment 2. (5%)

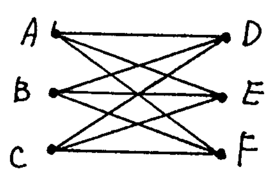
   (b) Calculate the probability of the event $\{$coin comes up heads, sum of dice $= 10\}$. (5%)

3. Solve the following recurrence relations.

   (a) $a_{n+2}^2 - 5a_{n+1}^2 + 6a_n^2 = 7n$, $n \geq 0$, $a_0 = a_1 = 1$. (5%)

   (b) $a_n + n\,a_{n-1} = n!$, $n \geq 1$, $a_0 = 1$. (5%)

4. Show that if the edge AD is removed from the following graph, the resulting graph is planar. (10%)



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5. (a) Rewrite the monstrosity $(((((01)1)(01)) + (((01)1)(01))))^*$ without using so many parentheses. Describe the language it represents. (5%)

   (b) Find a regular expression that represents the language consisting of the strings ϵ, 01, 0101, 010101, 01010101, $\cdots$. (5%)