Part I : Linear Algebra (50%)

1. Let A, B, C and D be vectors in 3–dimensional space. What is the geometric relation between the located vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$.

   (1) A = (1, −1, 3), B = (2, 4, 1),
   C = (4, −2, 5), D = (5, 3, 3). (3%)

   (2) A = (3, 3, π), B = (π+3, 5, π+1),
   C = (−2,2π+1, 95), D = (0, π+1, 95). (4%)

   (3) A = (−1, 1, −5), B = (2, −3, 4),
   C = (3, 1, 1), D = (−3, 9, −17). (3%)

2. Let V be a vector space over the field K. Let U and W be subspaces of V. Prove that the sum of U and W (i.e., U+W) forms a subspace of V. (10%)

3. Let f: S –> S' be a map which is both injective and surjective. Show that f has an inverse mapping. (10%)

4.

   (1) Let V be a vector space over the field K and let
   $A : V \to V$
   be an operator of V. Define the eigenvector and eigenvalue of A. (5%)

   (2) Let

$$A = \begin{pmatrix} a_1 & & \\ & a_2 & 0 \\ & 0 & \ddots & a_n \end{pmatrix}$$

   be a diagonal matrix in K.
   Find an eigenvector and an eigenvalue of A. (5%)

5. Define the following terms.

   (1) Linear mapping. (5%)

   (2) Bilinear mapping. (5%).

Part II : Discrete Mathematics (50%)

1. Find the number of ways of distributing 8 apples and 6 oranges to 3 children so that each child can get at least 2 apples and at most 2 oranges. (10%)

2. Consider the graph G = (V,E) where V = {1,2,3,4,5,6} and E = {(1,2), (2,3), (3,4), (4,5), (5,6), (1,6), (2,6), (5,2)}.
   (a) Find a simple path from 1 to 6 using five arcs. (2%)
   (b) Use the adjacency matrix of G to determine the number of 2-paths from 2 to 4. (4%)
   (c) Find the reachability matrix R of G. (4%)

3. Use the prefix code A = 000, B = 001, C = 01, D = 10, E = 111, and R = 110 to decode the following words :
   (a) 00000111000001000100000011100011 (5%)
   (b) 11111011001010100010001101010111 (5%)

4. Find a formula for the value of a $1000 saving certificate after n years if it pays 8% annual interest (compounded annually). How long does it take for the value of the certificate to double ? (10%)

5. A nickel, a dime, and a quarter are in a hat. We carry out the process of selecting first one coin and then a second.
   (a) Draw the rooted tree that illustrates this process, assuming we put the first coin back before drawing the second. (5%)
   (b) Draw the rooted tree that illustrates this process, assuming we do not put the first coin back before drawing the second. (5%)