86 學年度 國立成功大學 碩士班招生考試 資讯工程研究所 (甲,乙,丙) 計算机數學 試題

part I : Linear Algebra (50%)

1. (a) Find a basis for the nullspace of 5%

$$A=\begin{bmatrix}1 & 0 & 2\\ 1 & 1 & 4\end{bmatrix}$$

   (b) Verify that the basis found in (a) is orthogonal to the row space of A. 5%

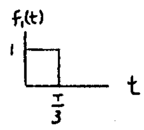
   (c) Given $\bar{x}=(3,3,3)^T$, (T denotes transpose), split it into a row space component $\bar{x}_r$ and a nullspace component $\bar{x}_n$. 5%

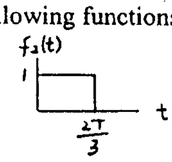
2. (a) Give the definition of an orthogonal matrix. 3%

   (b) Show that multiplication by an orthogonal matrix Q preserves lengths, that is, $\|Q\bar{x}\|=\|\bar{x}\|$ for every vector $\bar{x}$. 3%

   (c) Show that multiplication by an orthogonal matrix Q also preserves inner products, that is, $<Q\bar{x},Q\bar{y}>=\bar{x}^T\bar{y}$ for every $\bar{x}$ and $\bar{y}$. 4%

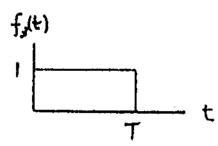
3. (a) Use the Gram-Schmidt orthogonalization procedure to find an orthonormal basis $\{\phi_1(t),\phi_2(t)\}$ for the set of the following functions $f_1(t), f_2(t)$.





   Plot $\phi_1(t)$ and $\phi_2(t)$. The inner product between $f_i(t)$ and $\phi_j(t)$ is defined by $<f_i(t),\phi_j(t)>=\int_0^T f_i(t)\phi_j(t)dt$. 5%

   (b) Find the function in the space spanned by $\{\phi_1(t),\phi_2(t)\}$ to best approximate (the one with least square error) the following function $f_3(t)$. 5%



   (c) Calculate the least square error in (b). 5%

4. Each year 20% of the students outside information engineering major move in, and 10% of the students inside move out. Assume that the total number of students remains the same every year.

   (a) Eventually (after infinite years), how many percentages of the total students remain outside the information engineering major (you can guess). 5%

   (b) Formulate the above problem in a matrix form, then solve the eigenvalue problem to obtain your answer in (a). 5%

(背面仍有題目,請繼續作答)

Part II Discrete Mathematics (50%)

1. Solve the following recurrence relations. (10%)
   (a) $a_{n+2}^2 - 5a_{n+1}^2 + 6a_n^2 = 7n, \quad n \ge 0, \; a_0 = a_1 = 1$
   (b) $a_n + na_{n-1} = n!, \quad n \ge 1, \; a_0 = 1$

2. ***Determine*** and ***explain*** whether each of the following statements is true. (10%)
   (a) Every group has at least one proper subgroup.
   (b) A group with only three elements is commutative.

3. A computer program selects an integer in $\{n : 1 \le n \le N\}$ at random and prints the result. This is repeated N times. (10%)
   (a) Set $N = 3$. What is the probability that the value n=1 appears in the printout at least once?
   (b) Set $N = 1{,}000{,}000$. What is the probability that the value n=1 appears in the printout at least once?

4. Construct a graph with vertex set $\{0,1\}^3$, which consists of 3-tuples of 0's and 1's, and with an edge between vertices v and w if v and w differ in exactly two coordinates. (10%)
   (a) Sketch the graph.
   (b) How many vertices does the graph have of each degree?

5. An urn has 4 yellow balls and 1 green ball. (10%)
   (a) Balls are drawn from the urn without replacement until the green ball is obtained. What is the probability for getting the green ball within 3 times?
   (b) Repeat part (a) if each ball is replaced after each drawing.