

1. Check if the following matrices are diagonalizable. If diagonalizable, compute a matrix that diagonalizes the given matrix. (10%)

(a)
$$\begin{bmatrix} 8 & -7 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

(b)
$$\begin{bmatrix} -7 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ -4 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

2. In the following, find the inverse of the matrix or else show that this matrix is singular. (10%)

(a)
$$\begin{bmatrix} -1 & 1 & 16 & 2 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 1 & 6 \\ 0 & 1 & 1 & -3 \end{bmatrix}$$

(b)
$$\begin{bmatrix} -2 & 6 & 0 & 0 \\ 1 & 4 & 4 & 11 \\ 4 & -4 & -5 & 3 \\ -3 & 1 & 2 & -6 \end{bmatrix}$$

3. Find the general solution of the system or show that the system has no solution. (10%)

$$8x_2 - 4x_3 + 10x_6 = 1$$

$$x_3 + x_5 - x_6 = 2$$

$$x_4 - 3x_5 + 2x_6 = 0$$

4. Show that **A** is row equivalent to **B** and **B** is row equivalent to **C**. Prove that **A** is row equivalent to **C**. (10%)
5. Determine the dimension of the subspace of R^3 consisting of all vectors parallel to a given plane through the origin. (Here, R^3 denotes a vector space with three components.) (10%)

(背面仍有題目,請繼續作答)

Part II Discrete Mathematics (50%)

- (12%) Which of the following describe equivalence relations? Justify your answer.
 - $L_1 \parallel L_2$ for straight lines in the plane if L_1 and L_2 are parallel.
 - $L_1 \perp L_2$ for straight lines in the plane if L_1 and L_2 are perpendicular.
 - $P_1 \triangleleft P_2$ for Americans if P_1 and P_2 live in the same state.
 - $P_1 \triangleright P_2$ for people if P_1 and P_2 have a parent in common.
- (12%) An urn contains three red and four black balls. A set of three balls is removed at random from the urn (without replacement). Give the probabilities that the three balls are
 - all red
 - all black
 - one red and two black
 - two red and one black
- (12%) The following table gives the binary operation for a group $(G, *)$ with elements a, b, c, d, e, f .

*	e	a	b	c	d	f
e	e	a	b	c	d	f
a	a	b	e	d	f	c
b	b	e	a	f	c	d
c	c	f	d	e	b	a
d	d	c	f	a	e	b
f	f	d	c	b	a	e

- List the members of the subgroup $\langle a \rangle$.
 - Show that $\langle a \rangle * c = c * \langle a \rangle$.
 - Describe the right cosets of $\langle d \rangle$.
- (14%) A 2-3 tree is a rooted tree such that each interior node has either two or three children and all paths from the root to the leaves have the same length.
 - How many types of 2-3 trees of height 2 are there?
 - Draw one tree of each type.