一、 Data Structure (50%)

1. (12%) Answer the following questions:

(a) Describe the characteristics of AVL tree.

(b) What is the complexity of time to delete a key from an AVL tree with N keys?

(c) Given a data set of N items, compare the number of comparisons for searching a single item when the data are stored in sorted array, link list and AVL-tree, respectively.

2. (15%) Answer the following questions

(a) How many comparisons are needed at least for sorting 5 elements by any comparison-based algorithm? Explain why.

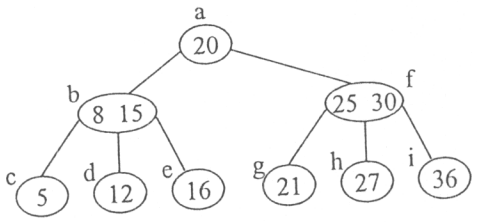
(b) Give the average time complexity of QUICKSORT, HEAPSORT and 2-WAY MERGE SORT, respectively.

(c) Please describe under what conditions you will use each of the sorting methods in (b) such that you can obtain best efficiency.

3. (15%)

(a) Give the definition of B-tree of order m.

(b) The following is a B-tree of order 3. Please plot the tree again after deleting key "20". How many disk accesses are needed?



4. (8%) Give an O(|V|)-time algorithm for determining whether or not a given undirected graph G = (V, E) contains a cycle. Explain why your algorithm's running time is independent of |E|.

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二. Algorithms (50%)

1. Consider the problem of evaluating a polynomial at a point. Given n coefficients $a_0, a_1, \ldots, a_{n-1}$ and a real number x, describe a $\theta(n)$-time algorithm to compute $\sum_{i=0}^{n-1} a_i x^i$. (10%)

2. (a) Explain why the statement, "The running time of algorithm A is at least $O(n^2)$," is content-free. (5%)
   (b) Is $2^{n+1}=O(2^n)$? Is $2^{2n}=O(2^n)$? (5%)

3. What is the largest k such that if you can multiply 3×3 matrices using k multiplications (not assuming commutativity of multiplication), then you can multiply $n \times n$ matrices in time $o(n^{\log 7})$? (20%)

4. Let (u, v) be a minimum-weight edge in a graph G. Does (u, v) belong to some minimum spanning tree of G? (10%)