Part I. Linear Algebra (50%)

1. (a) Find the sum of determinants. (5%)

$$\begin{vmatrix} 0 & 1 & 2 & 3 \\ -2 & -2 & 3 & 3 \\ 1 & 2 & -2 & -3 \\ 1 & 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \\ -1 & -1 & 4 & 4 \\ 2 & 3 & -1 & -2 \end{vmatrix}.$$

(b) Let A be a nonsingular $n \times n$ matrix with a nonzero cofactor A_{nn} at (n,n) entry and set $c = \det(A)/A_{nn}$. Show that if we subtract c from (n,n) entry of A then the resulting matrix will be singular. (10%)

2. Consider the inner product space $C[0,1]$ with inner product defined by

$$< f, g >= \int_0^1 f(x)g(x)dx$$

Let S be the subspace spanned by the vectors 1 and $2x-1$. Find the best least squares approximation to $\sqrt{x}$ by a function from the subspace S. (15%)

3. (a) Compute e^A for the following matrix. (10%)

$$A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

(b) Let L be the linear transformation mapping R^3 into R^3 defined by $L(\mathbf{x}) = A\mathbf{x}$, where $A = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -2 \\ 2 & -1 & -1 \end{bmatrix}$, and let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$. Find the matrix representing L with respect to $[\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3]$. (10%)

(背面仍有題目,請繼續作答)

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Discrete Mathematics 2002

1. [20%]

(a) Find the sequence generated with the following generating function, $f(x)=\dfrac{1-2x+2x^2}{1-x-6x^2}$.

(b) Let $x[n]=\begin{cases}2^{-n} & ,n\geq 0\\ 0 & ,otherwise\end{cases}$. Define the recurrence relation for $y[n]$ as follows: $y[n]=x[n]+(\frac{1}{2})x[n-1]+(-\frac{1}{3})y[n-1]$, where $y[-1]=1$, and $y[n]=0, for\ n\leq -2$. Find $y[n]$ for $n\geq 0$

2. [10%] A binary sequence receiver is specified as follows:
   (a) It starts to receive data after 3 consecutive 0's are met.
   (b) It outputs '01' when the sequence being received is of odd parity.
   (c) It outputs '10' when the sequence being received is of even parity.
   (d) The receiving procedure stops when 3 consecutive 1's are met. The system returns to the state for the next receiving activity.

   Draw the state diagram for this receiver.

3. [10%]
   (a) Give the definitions of even numbers and odd numbers. Prove that the sum of an even number and an odd number is odd.
   (b) Give the definitions of even functions and odd functions. Prove that any function can be the sum of an even function and an odd function.

4. [10%]

   Four symbols (a,b,c,d) are given the following codes to represent each of them, respectively: {0,10,11,101}. If a sequence of codes is generated based on any arbitrary combinations of the above symbols, is the sequence uniquely decodable? Prove your answer.